

# Reinforcement learning and human behavior

## Hanan Shteingart<sup>1</sup> and Yonatan Loewenstein<sup>1,2,3,4</sup>

The dominant computational approach to model operant learning and its underlying neural activity is model-free reinforcement learning (RL). However, there is accumulating behavioral and neuronal-related evidence that human (and animal) operant learning is far more multifaceted. Theoretical advances in RL, such as hierarchical and model-based RL extend the explanatory power of RL to account for some of these findings. Nevertheless, some other aspects of human behavior remain inexplicable even in the simplest tasks. Here we review developments and remaining challenges in relating RL models to human operant learning. In particular, we emphasize that learning a model of the world is an essential step before or in parallel to learning the policy in RL and discuss alternative models that directly learn a policy without an explicit world model in terms of state-action pairs.

### Addresses

<sup>1</sup> Edmond and Lily Safra Center for Brain Sciences, The Hebrew University, Jerusalem 91904, Israel

<sup>2</sup> Department of Neurobiology, The Alexander Silberman Institute of Life Sciences, The Hebrew University, Jerusalem 91904, Israel

<sup>3</sup> Department of Cognitive Science, The Hebrew University, Jerusalem 91904, Israel

<sup>4</sup> Center for the Study of Rationality, The Hebrew University, Jerusalem 91904, Israel

Corresponding author: Loewenstein, Yonatan ([yonatan@huji.ac.il](mailto:yonatan@huji.ac.il))

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### Model-free RL

The computational problem in many operant learning tasks can be formulated in a framework known as *Markov Decision Processes* (MDP) [1]. In MDPs, the world can be in one of several *states*, which determine the consequences of the agent's *actions* with respect to the future *rewards* and world states. A *policy* defines the agent behavior at a given situation. In MDPs, a policy is a mapping from the states of the environment to actions to be taken when in those states [1]. Finding the optimal *policy* is difficult because actions may have both immediate and long-term consequences. However, this problem can be simplified by estimating *values*, the expected cumulative (discounted) rewards

associated with these states and actions and using these values to choose the actions (for a detailed characterization of the mapping from values to actions in humans, see [2<sup>\*</sup>]).

Model-free RL, as its name suggests, is a family of RL algorithms devised to learn the values of the states without learning the full specification of the MDP. In a class of model-free algorithms, known as temporal-difference learning, the learning of the values is based on the *reward-prediction error* (RPE), the discrepancy between the expected reward before and after an action is taken (taking into account also the ensuing obtained reward). The hypothesis that the brain utilizes model-free RL for operant learning holds considerable sway in the fields of neuroeconomics. This hypothesis is supported by experiments demonstrating that in primates, the phasic activity of mid-brain dopaminergic neurons is correlated with the RPE [3,4]. In mice, this correlation was also shown to be causal: optogenetic activation of dopaminergic neurons is sufficient to drive operant learning, supporting the hypothesis that the dopaminergic neurons encode the RPE, which is used for operant learning [5<sup>\*</sup>]. Other putative brain regions for this computation are the striatum, whose activity is correlated with values of the states and/or actions [6,7] and the nucleus accumbens and pallidum, which are involved in the selection of the actions [8]. In addition to its neural correlates, model-free RL has been used to account for the trial-by-trial dynamics (e.g., [2<sup>\*</sup>]) and for several robust aggregate features of human behavior such as risk aversion [9], recency [10] and primacy [2<sup>\*</sup>]. Moreover, model-free RL has been proven useful in the field of computational psychiatry as a way of diagnosing and characterizing different pathologies [11–13,14<sup>\*</sup>].

However, there is also evidence that the correspondence between dopaminergic neurons and the RPE is more complex and diverse than was previously thought [15]. First, dopaminergic neurons increase their firing rate in response to both surprisingly positive and negative reinforcements [16,17]. Second, dopaminergic activity is correlated with other variables of the task, such as uncertainty [18]. Third, the RPE is not exclusively represented by dopamine, as additional neuromodulators, in particular serotonin, are also correlated with the RPE [19]. Finally, some findings suggest that reinforcement and punishment signals are not local but rather ubiquitous in the human brain [20]. These results challenge the dominance of the anatomically modular model-free RL as a model for operant learning.

## Model-based RL

When training is intense, task-independent reward devaluation, for example, through satiety, has only a little immediate effect on behavior. This habitual learning is consistent with model-free RL because in this framework, the value of an action is updated only when it is executed. By contrast, when training is moderate, the response to reward devaluation is immediate and substantial [21]. This and other behaviors (e.g., planning) are consistent with an alternative RL approach, known as *model-based* RL, in which a model of the world, that is, the parameters that specify the MDP are learned before choosing a policy. The effect of reward devaluation after moderate training can be explained by model-based RL because a change in a world parameter (e.g., the devaluation of the reward as a result of satiety) can be used to update (off-line) the values of other states and actions.

If the parameters of the MDP are known, one can compute the values of all states and actions, for example by means of dynamic programming or Monte-Carlo simulation. Alternatively, one could choose an action by expanding a look-ahead decision tree on-line [1]. However, a full expansion of a look-ahead tree is computationally difficult because the number of branches increases exponentially with the height of the tree, so pruning of the tree is a necessary approximation. Indeed, a recent study has suggested that humans prune the decision-tree, by trimming branches associated with large losses [14\*].

Whether or not model-based and model-free learning are implemented by two anatomically distinct systems is a subject of debate. In support of anatomical modularity are findings that the medial striatum is more engaged during planning whereas the lateral striatum is more engaged during choices in extensively trained tasks [22]. In addition, the *state prediction error*, which signals the discrepancy between the current model and the observed state transitions is correlated with activity in the intraparietal sulcus and lateral prefrontal cortex, spatially separated from the main correlate of the RPE in the ventral striatum [23]. Findings that negate anatomical modularity include reports of signatures of both model-based and model-free learning in the ventral striatum [24\*].

## The curse of dimensionality and the blessing of hierarchical RL

There are theoretical reasons why the RL models described above cannot fully account for operant learning in natural environments. First, the computational problem of finding the values is bedeviled by the ‘curse of dimensionality’: the number of states is exponential with the number of variable, which define a state [1]. Second, when the state of the world is only partially known (i.e., the environment is a partially observable MDP

(POMDP), applying model-free algorithms such as Q-learning may converge to a solution that is far from optimality or may fail to converge altogether [25]. One approach to addressing these problems is to break down the learning task into a hierarchy of simpler learning problems, a framework known as *Hierarchical Reinforcement Learning* (HRL) [26]. Neuroimaging studies have indeed found neural responses that are consistent with subgoal-related RPE, as is predicted by HRL [27\*].

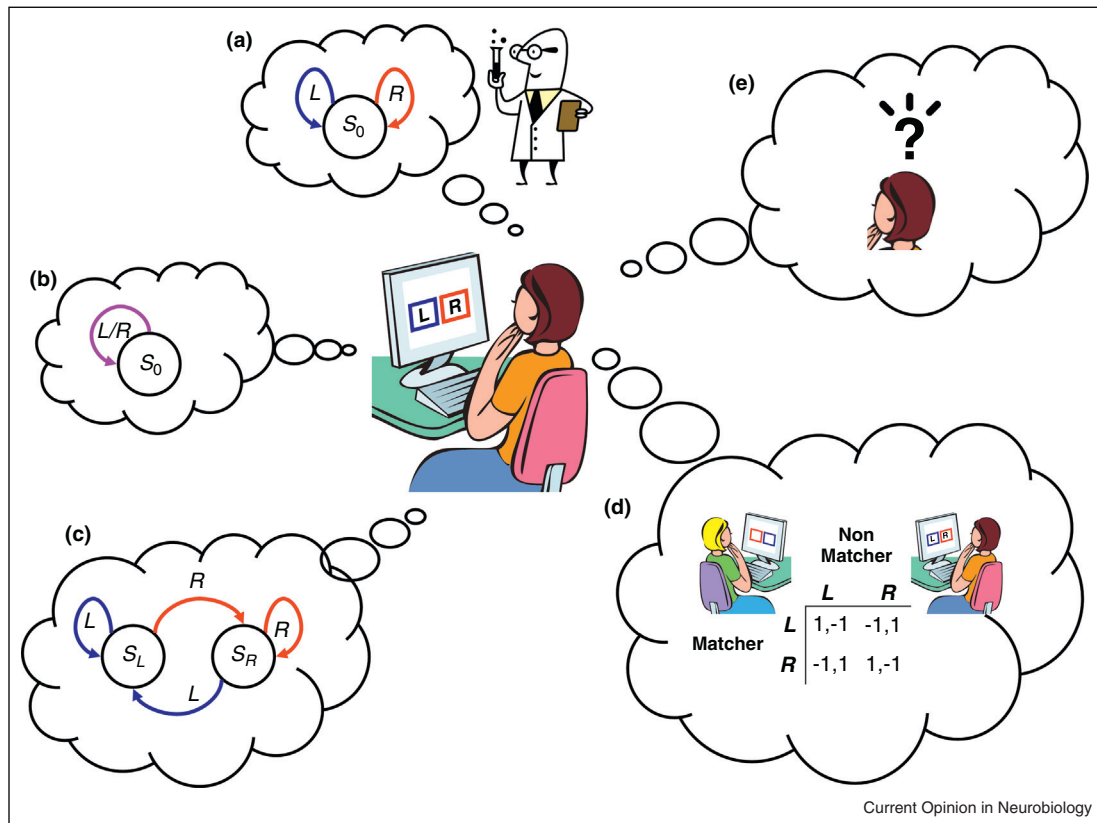
## Challenges in relating human behavior to RL algorithms

Despite the many successes of the different RL algorithms in explaining some of the observed human operant learning behaviors, others are still difficult to account for. For example, humans tend to alternate rather than repeat an action after receiving a positively surprising payoff. This behavior is observed both in simple repeated two-alternative force choice tasks with probabilistic rewards (also known as the 2-armed bandit task, Figure 1a) and in the stock market [28]. Moreover, a recent study found that the behavior of half of the participants in a 4-alternative version of the bandit task, known as the Iowa gambling task, is better explained by the simple ad-hoc heuristic ‘win-stay, lose-shift’ (WSLS) than by RL models [29\*]. Another challenge to the current RL models is the tremendous heterogeneity in reports on human operant learning, even in simple bandit tasks, measured in different laboratories in slightly different conditions. For example, as was described above, the WSLS observed in 4-arm bandit [29\*] is inconsistent with the alternation after positively surprising payoffs discussed above [28]. Additionally, *probability matching*, the tendency to choose an action in proportion to the probability of reward associated with that action, has been a subject of debate over half-a-century. On one hand, there are numerous reports supporting this law of behavior both in the laboratory and when humans gamble substantial amounts of money on the outcome of real-life situations [30]. On the other hand, there is abundant literature arguing that people deviate from probability matching in favor of choosing the more rewarding action (maximization) [31]. Finally, there is substantial heterogeneity not only between subjects and laboratories but also within subjects over time. A recent study has demonstrated substantial day-to-day fluctuations in the learning behavior of monkeys in the two-armed bandit task and has shown that these fluctuations are correlated with day-to-day fluctuations in the neural activity in the putamen [32].

## Heterogeneity in world model

The lack of uniformity regarding behavior even in simple tasks could be due to heterogeneity in the prior expectations of the participants. From the experimentalist point of view, the two-armed bandit task, for example, is simple: the world is characterized by a single state and two actions (Figure 1a). However, from the participant

Figure 1



Repertoire of possible world models. In this example, a participant is tested in the two-armed bandit task. **(a)** From the experimentalist's point of view (scientist caricature), the world is characterized by a single state ( $S_0$ ) and two actions: left (blue,  $L$ ) or right (red,  $R$ ) button press. However, from the participant's point of view there is an infinite repertoire of possible world models characterized by different sets of states and actions. **(b)** With respect to the action sets, she may assume that there is only a single available action, pressing any button, regardless of its location (purple,  $L/R$ ). **(c)** With respect to the state sets, the participant may assume that the state is defined by her last action ( $S_L$  and  $S_R$ , for previous  $L$  and  $R$  action, respectively). **(d)** Moreover, the participant may assume she is playing a penny-matching game with another human. **(e)** These and other possible assumptions may lead to very different predictions in the framework of RL.

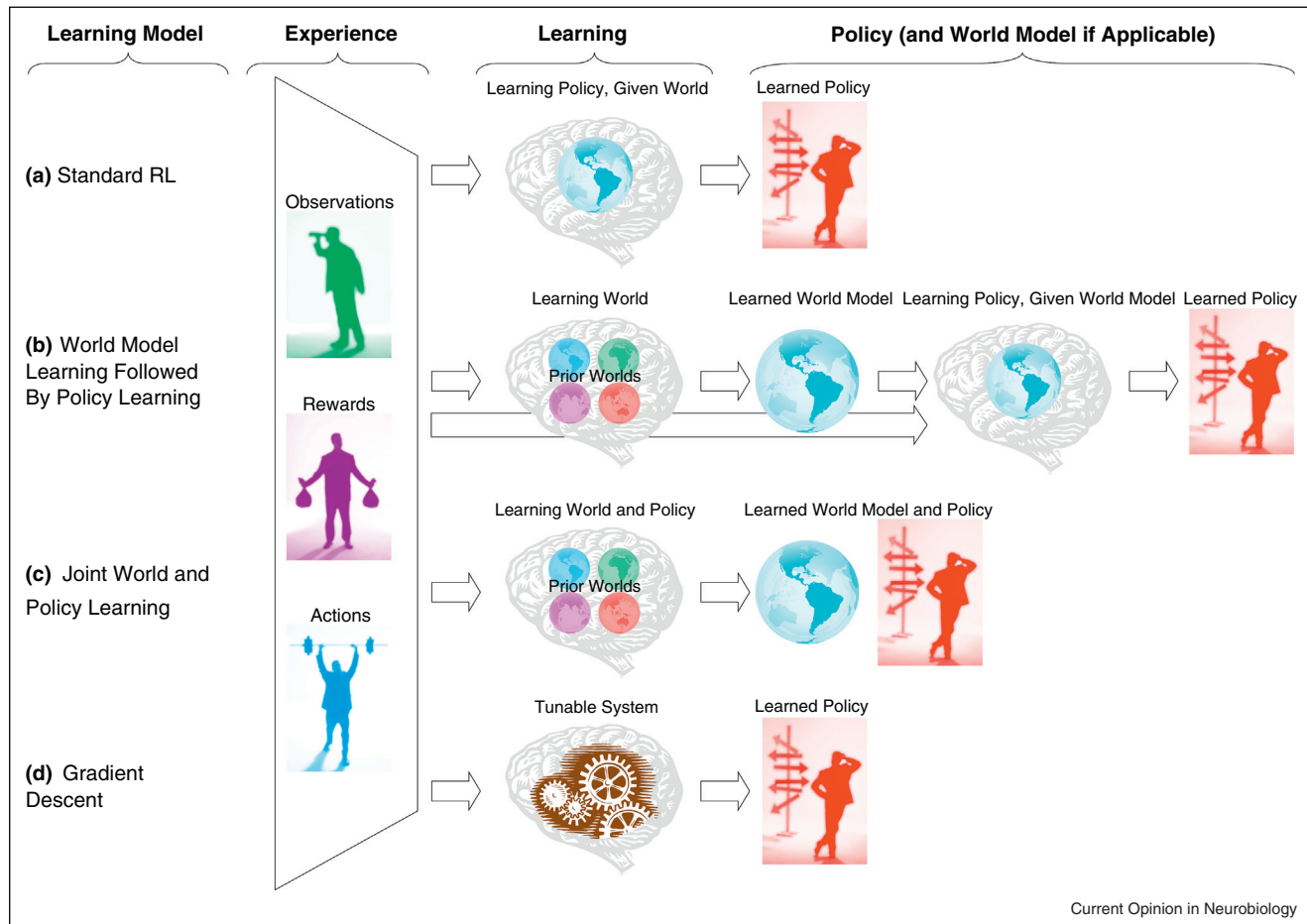
point of view there is, theoretically, an infinite repertoire of possible world models characterized by different sets of states and actions. This could be true even when precise instructions are given due to, for example, lack of trust, inattention or forgetfulness. With respect to the actions set, the participant may assume that there is only a single available action, the pressing of any button, regardless of its properties (Figure 1b). Alternatively, differences in the timing of the button press, the finger used, among others, could all define different actions. Such precise definition of action, which is irrelevant to the task, may end with non-optimal behavior [32]. With respect to the states set, the participant may assume that there are several states that depend on the history of actions and/or rewards. For example, the participant may assume that the state is defined by the last action (Figure 1c), the last action and the last reward, or a function of the long history of actions [33]. Finally, the participant may assume a strategic game setting such as a matching-pennies game (Figure 1d). These and other possible assumptions (Figure 1e) may

lead to very different predictions on behavior [34]. In support of this possibility, experimental manipulations such as instructions, which are irrelevant to the reward schedule, but may change the prior belief about the number of states can have a considerable effect on human behavior [35]. Finally, humans and animals have been shown to develop idiosyncratic and stereotyped superstitious behaviors even in simple laboratory settings [36]. If participants fail to recognize the true structure of a learning problem in simple laboratory settings, they may also fail to identify the relevant states and actions when learning from rewards in natural environments. For example, professional basketball players have been shown to overgeneralize when learning from their experience [37].

### Learning the world model

Many models of operant learning often take as given that the learner has already recognized the available sets of states and actions (Figure 2a). Hence, when attempting to

Figure 2



Alternative models of operant learning. In operant learning, experience (left trapezoid), composed of present and past observations, actions and rewards, is used to learn a policy. **(a)** Standard RL models typically assume that the learner (brain gray icon) has access to the relevant states and actions set (represented by a bluish world icon) before the learning of the policy. Alternative suggestions are that the state and action sets are learned from experience and from prior expectations (different world icons) before **(b)** or in parallel **(c)** to the learning of the policy. **(d)** Alternatively, the agent may directly learn without an explicit representation of states and actions, but rather by tuning a parametric policy (cog wheels icon), for example, using stochastic gradient methods on this policy's parameters.

account for human behavior they fail to consider the necessary preliminary step of identifying them (correctly or incorrectly). In machine learning, classification is often preceded by an unsupervised dimension-reduction for feature extraction [38,39]. Similarly, it has been suggested that operant learning is a two-step process (Figure 2b): in the first step, the state and action sets are learned from the history (possibly using priors on the world), where in the second step RL algorithms are utilized to find the optimal policy given these sets [40]. An interesting alternative is that the relevant state-action sets and the policy are learned in parallel (Figure 2c). For example, in a new approach in RL, known as *feature RL*, the state set and the values of the states are learned simultaneously from the history of observations, actions and rewards. One crucial property of feature RL is that it neither requires nor learns a model of the complete observation space, but rather

learns a model that is based on the reward-relevant observations [41].

### Learning without states

Operant learning can also be accomplished without an explicit representation of states and actions, by directly tuning a parametric policy (Figure 2d). A plausible implementation of such direct policy learning algorithms is using stochastic policy-gradient methods [42–44]. The idea behind these methods is that the gradient of the average reward (with respect to policy parameter) can be estimated on-line by perturbing a neural network model with noise and considering the effect of these perturbations on the stream of payoffs delivered to the learning agent. Changes in the policy in the direction of this estimated gradient are bound, under certain assumptions, to improve performance. However, local minima may



prevent the learning dynamics from converging to the optimal solution.

Direct policy methods have been proposed to explain birdsong learning [45] and have received some experimental support [46,47]. In humans, a model for gradient learning in spiking neurons [48,49] has been shown to be consistent with the dynamics of human learning in two-player games [50]. Under certain conditions, gradient-like learning can be implemented using covariance-based synaptic plasticity. Interestingly, operant matching (not to be confused with probability matching) naturally emerges in this framework [51,52]. A model based on attractor dynamics and covariance-based synaptic plasticity has been shown to quantitatively account for free operant learning in rats [53]. However, the experimental evidence for gradient-based learning, implemented at the level of single synapses, awaits future experiments.

### Concluding remarks

RL is the dominant theoretical framework to operant learning in humans and animals. RL models were partially successful in quantitative modeling of learning behavior and provided important insights into the putative role of different brain structures in operant learning. Yet, substantial theoretical as well as experimental challenges remain, indicating that these models may be substantially oversimplified. In particular, how state-space representations are learned in operant learning remain important challenges for future research.

### References and recommended reading

Papers of particular interest, published within the period of review, have been highlighted as:

- of special interest
- of outstanding interest

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