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Thin element approximation for the analysis of blazed gratings: simplified model and validity limits

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Abstract

The thin element approximation is widely used to predict the diffraction efficiency of thin periodic diffractive optical elements (DOEs). However, as the period-to-wavelength ratio is reduced, the approximation becomes inaccurate. A model based on a ‘‘shadow concept’’ can be used to predict the diffraction efficiency with high accuracy. Hereby we extend the model to include the effect of multi-level staircase structures and non-perpendicular incident angles. We also present an error map and define regions of validity for the thin element approximation (TEA) and the shadow model.

Keywords: Diffraction; Blazed gratings

1. Introduction

The thin element approximation (TEA) is commonly used for the design and the analysis of diffractive optical elements (DOEs). The approach is valid as long as the feature size is large compared with the wavelength, and the element thickness is comparable with the wavelength. However, the validity breaks down in cases where the period-length to wavelength ratio decreases, or when the element is too thick. In such cases, rigorous analysis (for cases of periodic elements), or numerical approaches should be used [1–6]. For periodic structures, one frequently uses the rigorous coupled wave analysis (RCWA) [1]. The approach, which is popular within the optics community, provides a solution for the wave equation by using a plane wave decomposition of the dielectric constant and by assuming a periodic solution. Although an infinite number of periods is assumed, the results are sufficiently accurate even for the case of few tens of periods.

The RCWA approach, although valid for any ‘‘period-length to wavelength’’ ratio, is practically limited to the analysis of high-resolution gratings. When the period-to-wavelength ratio is large, many propagating diffraction orders exist and all should be taken into account, so that massive computational efforts are needed. On the other hand, if the TEA could be used, results would be obtained immediately. Thus, the importance of

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defining validity regions for the TEA by estimating the error with respect to grating parameters such as period, thickness, duty ratio, number of phase levels, refractive index and angle of incidence is clear. Several authors [7,8] addressed this issue.

Since TEA breaks down as the period-to-wavelength ratio decreases, it would be desirable to develop an approximate model, capable to predict the diffraction efficiency of DOEs with high accuracy, even for moderate period-to-wavelength ratios. An example for such a need is given in [9], where arrays of Fresnel lenses are discussed. The deviation from the TEA is specifically mentioned there, while claiming that for lenses of the size presented, more accurate analysis would require enormous computing power.

During the years, several models for the prediction of diffraction efficiency based on the "thin" Born approximation [10] and on the so-called "shadow effect" [11–15] were suggested. Indeed, the shadow model shows improved accuracy with respect to the TEA. However, it cannot accurately predict the diffraction efficiency of a multi-level DOE, in particular for cases where only few phase levels exist. Moreover, the proposed shadow model equations are given only for perpendicular incident illumination angle.

The motivation of this manuscript is to provide the reader with the tools to estimate the error caused by using the TEA and to offer an improved and extended model capable of calculating the diffraction efficiency more accurately. Throughout the manuscript, a multi-level dielectric grating is used as a test case. The diffraction efficiency of a multi-level grating as a function of the period-length to wavelength ratio, the number of phase levels and the modulation depth was calculated, and compared with the values predicted by the TEA. Although some of the data were published in the literature, we present these data in a more useful fashion, whereby an error map showing the relative error vs. the period to wavelength ratio as well as the grating thickness is computed. Moreover, the validity regions for the various models are calculated. We present in this work an extended version of the "shadow model" approximation, taking into account additional terms affecting the diffraction efficiency, particularly for the case of only few phase levels (e.g. binary grating). The extended model can also cope with non-perpendicular incident angles. Based on the extended model, an equation for the diffraction efficiency is derived, enabling fast calculation of the diffraction efficiency for the case of multi-level blazed grating. The equation predicts the results with high accuracy for regions where the TEA breaks down, as long as the period length is at least a few wavelengths.

In Section 2 a comparison between the accurate RCWA results and the TEA prediction is given. The shadow model is discussed in Section 3. Section 4 presents a refinement of the shadow model, mainly useful for the unique case of a blazed grating having only few phase levels. Section 5 discusses the results and concludes the manuscript.

2. RCWA and thin element approximation comparison

Based on the TEA, the diffraction efficiency of the diffraction order m diffracted by a multi-level blazed grating with N phase levels, each providing an additional phase delay of $2\pi K/N$ is given by [12,16]

$$\eta(N,m,k) = \left[\frac{\sin(\pi m/N)}{\pi m}\,\frac{\sin[\pi(m-K)]}{\sin[\pi(m-K)/N]}\right]^2. \tag{1}$$

It can be shown that non-vanishing diffraction efficiency terms exist only if $(m-K)/N = q$, where q is an integer. For m, K fulfilling this requirement, $\{\sin[\pi(m-K)]\}/\{\sin[\pi(m-K)/N]\} = N$. As a result, the diffraction efficiency can be rewritten as

$$\eta(N,m) = \left\{\begin{matrix} \mathrm{sinc}(m/N), & (m-K)/N = q, \\ 0, & \text{elsewhere}. \end{matrix}\right\}. \tag{2}$$

The maximum diffraction efficiency occurs for $m = K$, i.e. when the diffraction and the refraction conditions are matched. For this reason, the letter "m" will be used from now on both for diffraction order as well as for phase modulation depth, since our attention is addressed only towards maximum diffraction efficiency issues.

In order to account also for the interface intensity reflection loss, Eq. (2) should be multiplied by the Fresnel transmission coefficient, $4n/(n+1)^2$, where n is the refractive index of the material, and normal incident angle is assumed. In addition, it is assumed that only one boundary exists (between the grating structure and the air). The other boundary (between the substrate and the air) was eliminated by assuming an identical refractive index for the DOE and for the adjacent medium. This assumption would also hold if an AR coating is applied on the substrate.

The efficiency predicted by the TEA (Eq. (2)) is obtained by assuming that the field beyond the DOE is the multiplication of the incident field with the complex amplitude function of the DOE. However, such an approximation holds only if the period-to-wavelength ratio is large enough. If this is not the case, the exact Maxwell equations should be solved within the element and the field beyond it can be found by proper boundary conditions (for TE and TM polarization states). However, such exact calculation of the diffraction efficiency involves massive computation and cannot be expressed in a single closed equation. Thus, it is always preferable to utilize the approximate equation suggested before (Eq. (2)), if conditions permit. Obviously, before using Eq. (2) one must be familiar with its validity region, i.e. what are the parameters for which the above equation is in good agreement with the accurate RCWA results. We will base the criterion for "good agreement" on diffraction efficiency considerations.

In order to find the validity regions, the RCWA approach was used for calculating the diffraction efficiency of a multi-level blazed grating as a function of the ratio d/λ (where d is the grating period and λ is the wavelength). The calculation was performed for several values of N (where N is the number of phase levels), and for diffraction orders $m = 1, 2, 3, 4$. A refractive index of 1.5 was assumed for the substrate, so that the Fresnel transmission coefficient is 0.96. The obtained TE polarization diffraction efficiency can be seen in Fig. 1. The horizontal dashed lines represent the diffraction efficiency values predicted by the thin element approximation. The results are similar, although more detailed, to those that can be found in [7].

As can be seen from the above figures, the accurate RCWA efficiency curves converge towards the TEA values for large period-to-wavelength ratio, while for small (and even moderate) ratios, the diffraction efficiency deviates significantly from the TEA prediction, especially when the element is thick (i.e. large values of m).

We will now calculate the relative error ε between the accurate results and the results predicted by Eq. (2), following the definition:

$$\varepsilon(\%\text{error}) = 100\frac{|\eta_S(N,m) - \eta_R(N,m,\lambda/d)|}{\eta_S(N,m)}, \qquad (3)$$

where $\eta_R(N,m,\lambda/d)$ is the diffraction efficiency computed by the RCWA approach, d is the grating period, λ is the wavelength and η_S is the TEA result (Eq. (2)).

It is convenient to present the error results by an error map (see Fig. 2), which shows the relative error, in percentage points, as a function of m and d/λ (solid curves). The map has been calculated for the case of 16 phase levels ($N = 16$). As expected, the relative error increases with the increase of m and the decrease of d/λ. It can be noticed that the constant error curves are almost linear, and their slope increasing for higher error values, as also shown by the asymptotic dashed lines. The asymptotic behavior of these curves is not unexpected, and will be calculated in Section 3.

By observing the error map, it is clear that low error values can be obtained only for large d/λ ratios. As an example, for $m = 4$ an error below 10% will be obtained only if $d/\lambda > 23$. For such a case, almost 50 orders of diffraction can propagate. In addition, several evanescent orders should also be taken into account, so that accurate results will be obtained only by considering at least 70–80 orders. The importance of using an adequate number of orders was examined by simulating a test case, in which the diffraction efficiency of an 8-level blazed grating ($m = 2$, $d/\lambda = 40$) was calculated for a number of orders. The results are presented in Fig. 3. One notes that when the number of orders is too small, the diffraction efficiency oscillates. Although the magnitude of the oscillations is relatively small in this case, higher magnitude may be exhibited for other cases. Only

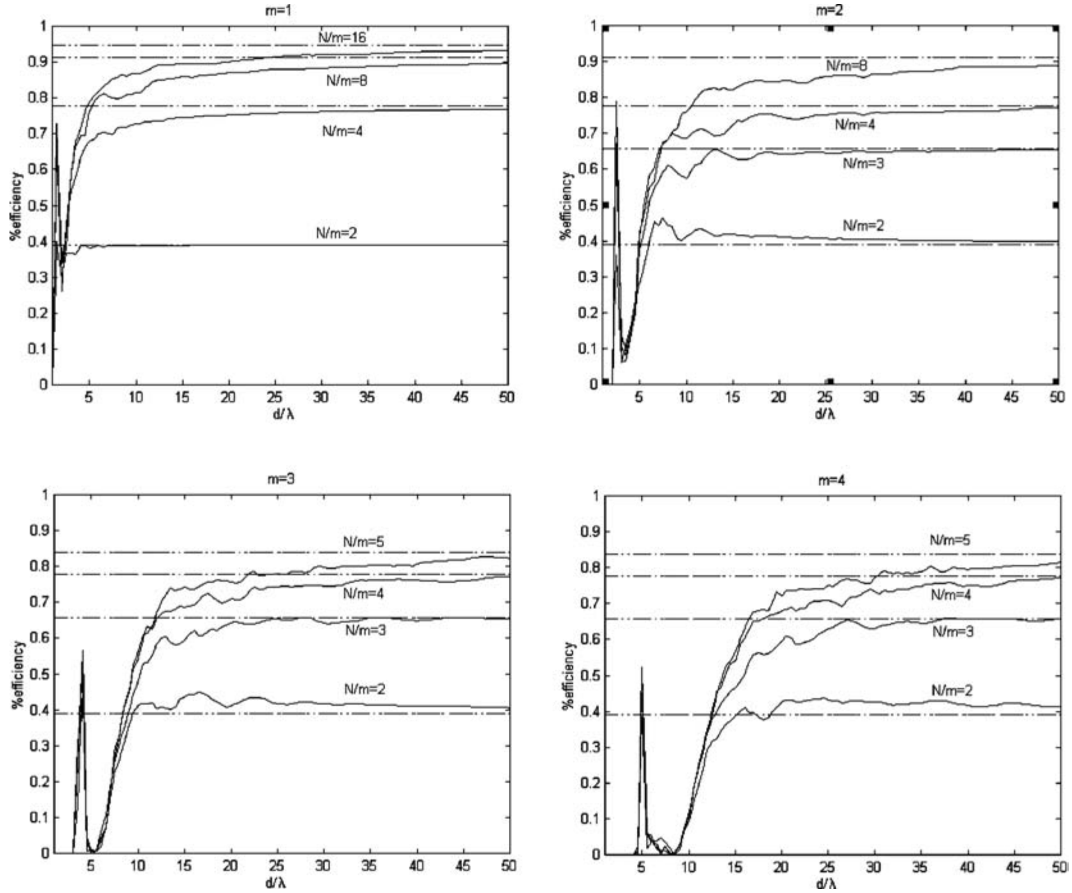

Fig. 1. Diffraction efficiency obtained using the RCWA approach vs. d/λ for various values of N, m. (dashed lines: the asymptotic prediction of the TEA).

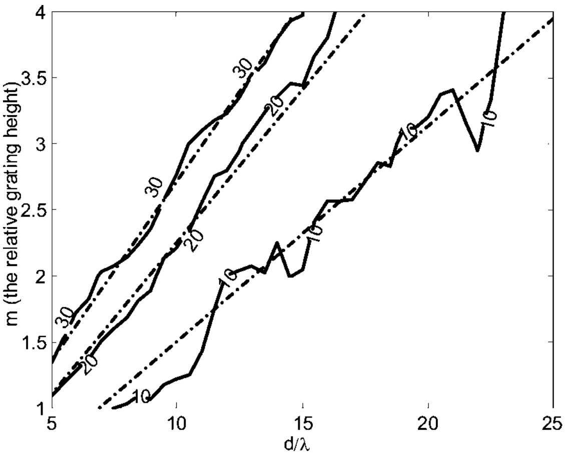

Fig. 2. The error map. Solid lines: results obtained by using Eq. (3); dashed lines: results obtained by using Eq. (17).

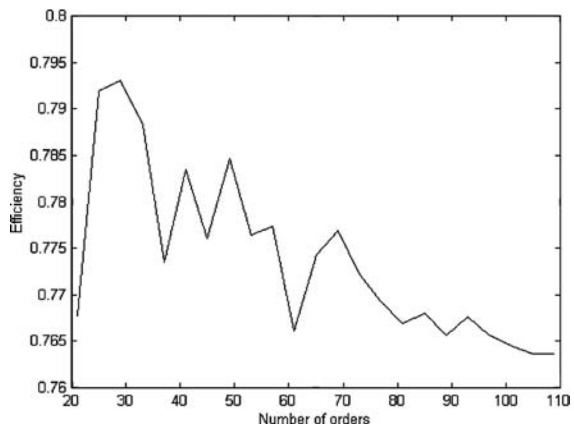

Fig. 3. Diffraction efficiency vs. number of orders ($N = 8,\ m = 2,\ d/\lambda = 40$).

by using a sufficiently high number of orders, the results stabilize and converge.

When many diffraction orders (say 100) are kept, the required matrices for calculating the field using the RCWA approach become large. Assuming 100 orders, there are 400 complex unknowns and the same number of equations, meaning that we have to invert a matrix of 400×400 complex elements. Although nowadays such computation can be easily performed, applying an optimization algorithm for the design of such DOEs based on the RCWA approach would still be a time consuming task. Thus, if a model capable to predict accurately the diffraction efficiency for moderate values of d/λ could be derived, it would provide significant advantages. Such a model is presented in the following section.

3. The shadow model

In this section we describe an extended TEA model, providing better prediction of the diffraction efficiency. The presented model is based on ray optics considerations. Normally, incident rays are assumed to propagate within the DOE without any deflection (extension to non-perpendicular incident angles is given later on). Deflection occurs at the outer surface, from which the rays emerge out of the DOE. The deflection angle of the outgoing rays is given by the grating equation.

According to the model, the diffraction efficiency predicted by the TEA is reduced due to two main reasons.

3.1. "Shadow" between two neighboring periods

This loss mechanism, which was suggested earlier [11–15], is sketched in Fig. 4. As can be seen, the emerging rays exhibit a "dead zone" (marked by the letter "V"), reducing the diffraction efficiency. A similar "dead zone" is exhibited also when the reverse case is analyzed, i.e., light incident from the non-flat surface. In that case, some rays are actually blocked, leading essentially to the same result. For first-order of diffraction, the ratio of the dead zone to the grating period is given by [11–14]

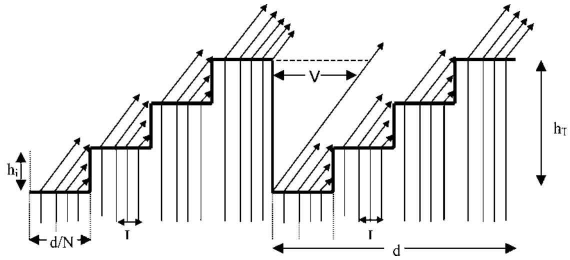

Fig. 4. The geometry of the suggested model.

$$\mathrm{SH}_1 = \frac{\lambda h_T}{d^2\sqrt{1-(\lambda/d)^2}}, \tag{4}$$

where SH_1 is the "inter period shadow". For the m's order of diffraction it can easily be shown that the inter period shadow is given by

$$\mathrm{SH}_1 = \frac{m\lambda h_T}{d^2\sqrt{1-(m\lambda/d)^2}}. \tag{5}$$

Here h_T is the grating depth, i.e.

$$h_T = \frac{N-1}{N}\frac{m\lambda}{(n-1)}. \tag{6}$$

By using the grating equation

$$d(\sin(\theta_{\mathrm{out}})) = m\lambda, \tag{7}$$

where θ_{out} is the angle of the outgoing ray (measured from the DOE normal), one obtains

$$\mathrm{SH}_1 = \frac{N-1}{N}\frac{m}{n-1}\frac{\lambda}{d}\tan(\theta_{\mathrm{out}}). \tag{8}$$

3.2. "Shadow" between two neighboring "stairs" (phase levels)

While some of the outgoing rays are free to propagate, others, coined "shadow rays", will hit a "barrier" which is the steep wall of the next phase level, as shown in Fig. 4. This intra-period loss is associated with the additional "shadow rays" that cannot propagate, thus further reducing the diffraction efficiency (for the specific order for which the grating was optimized for). To the best of our knowledge, this additional shadow factor has not been taken in consideration previously, most likely since the shadow analysis was derived for continuous structures initially.

Simple trigonometry indicates that an emerging ray will be blocked if its lateral location (measured from the left edge of the step) is larger than

$$T = h_i \tan(\theta_{\text{out}}), \tag{9}$$

where h_i is the height of each barrier, given by

$$h_i = \frac{h_T}{N} = \frac{m\lambda}{N(n-1)}. \tag{10}$$

We should note that only $N-1$ out of N steps are experiencing the shadow effect, since rays hitting the upper step are not obstructed. Therefore, the relative shadow area is given by

$$\text{SH}_2 = \frac{N-1}{N}\frac{T}{d/N} = \frac{N-1}{N}\frac{m}{n-1}\frac{\lambda}{d}\tan(\theta_{\text{out}}). \tag{11}$$

One readily sees that $\text{SH}_1 = \text{SH}_2$, i.e. the two shadow effects are quantitatively identical. As will be shown later, this equality does not hold for non perpendicular incident angles. The predicted diffraction efficiency is obtained by subtracting these two shadow sources from the TEA prediction

$$\eta\left(m, N, \frac{\lambda}{d}\right) = \frac{4n}{(n+1)^2}\left[\text{sinc}\left(\frac{m}{N}\right)\right]^2 \times \left(1 - 2\frac{N-1}{N}\frac{m}{n-1}\frac{\lambda}{d}\tan(\theta_{\text{out}})\right). \tag{12}$$

As can be seen from the above expression, the shadow effect is twice larger than that calculated in previous publications.

The shadow effect can be reduced by using a blazed-binary structure [11], whereby phase modulation is achieved by controlling the effective index rather than having a surface relief element. As a result, rays are gradually bent within the structure.

By comparing our model to accurate results, obtained by the RCWA approach, we estimate that Eq. (12) can be used with relatively high accuracy as long as the total efficiency does not drop to less than 50% of the original TEA prediction, i.e. as long as

$$\frac{N-1}{N}\frac{m}{n-1}\frac{\lambda}{d}\tan(\theta_{\text{out}}) < 0.5. \tag{13}$$

Upon substituting $\tan(\theta) \approx \sin(\theta)$ and applying Eq. (7) one gets

$$\frac{d}{\lambda} > m\sqrt{\frac{2(N-1)}{N(n-1)}}. \tag{14}$$

For a typical case of $N = 16$, $n = 1.5$ (glass) and $m = 2$, Eq. (12) is valid for $d/\lambda > 3.9$, whereas the TEA yield accurate results only for $d/\lambda > 15$ (assuming accuracy of $\sim$10%, see Fig. 2). If an accuracy of 5% is needed the above ratio increase towards $d/\lambda \sim 40$.

Figs. 5 and 6 demonstrate the higher accuracy obtained by using the "shadow model", in comparison to that of the TEA. Refractive index of 1.5 and perpendicular incident angle were assumed. The solid line represents the results obtained by using the RCWA approach, while the dashed lines were calculated according to the shadow model. One readily sees that the above model can be used as a good predictor for the diffraction efficiency. For the case of binary grating, or binary equivalent, defined by $N/m = 2$, the model is less accurate. This case will be discussed later. It is clear from these curves that as long as one seeks to evaluate the diffraction efficiency and good estimates rather than exact results are sufficient, the

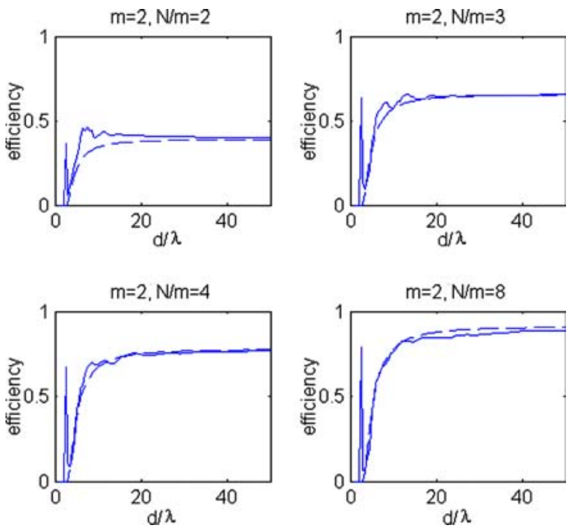

Fig. 5. Comparison of the RCWA and the shadow model: $m = 2$. Solid line: RCWA results. Dashed line: shadow model results.

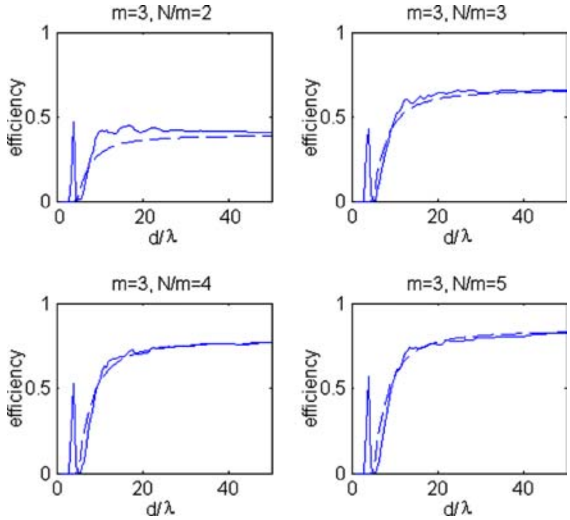

Fig. 6. Comparison of the RCWA and the shadow model: $m = 3$. Solid line: RCWA results. Dashed line: shadow model results.

RCWA approach can be replaced by the approximated simple equation (Eq. (12)), as long as the grating parameters are within the region of validity, defined in Eq. (14). We also note that the RCWA curve fits our present shadow analysis result (Eq. (12)), if the factor 2 is adopted.

From Eq. (12) it can be noticed that the shadow effect increases for smaller values of the refractive index n. This should be expected since for low index materials, thicker elements should be fabricated in order to achieve the same phase modulation, and thus additional portion of the diffracted rays is blocked. One would thus expect to obtain higher diffraction efficiency by using high index materials. However, at the same time the Fresnel reflection coefficient also increases due to the higher refractive index values. To examine these trade-offs, the diffraction efficiency versus the ratio of the period-length to wavelength was calculated using the RCWA approach for several values of the refractive index n. Values of $m = 3$ and $N = 15$ were assumed. The results, presented in Fig. 7, meet the expectations. For small period to wavelength ratios, the shadow effect is indeed dominant and diffraction efficiency increases with the increase of the refractive index. On the other hand, for large period-to-wavelength ratios, where the shadow effect is negligible, the Fresnel coefficient is dominant and the efficiency is reduced with

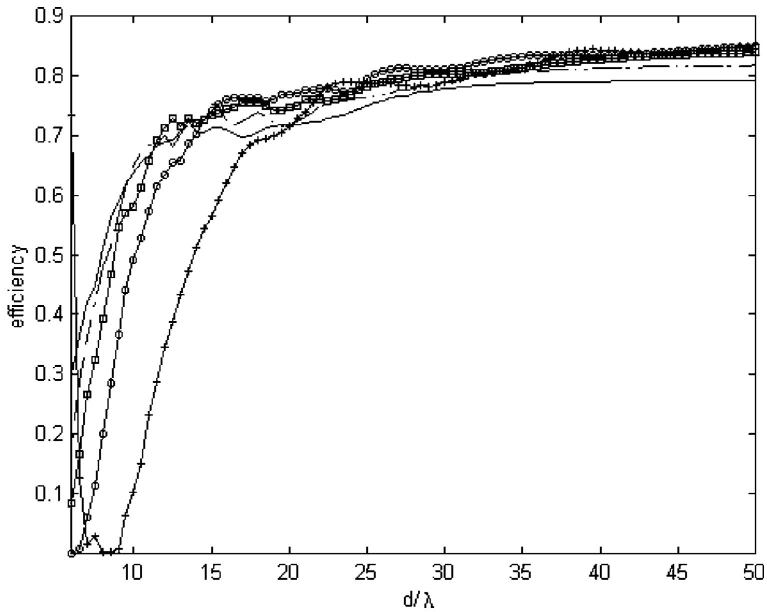

Fig. 7. Efficiency vs. d/λ for several values of refractive index. Plus: $n = 1.2$; circle: $n = 1.4$; square: $n = 1.6$; dashed: $n = 1.8$; solid: $n = 2$. Calculation made for the case of $m = 3$ and $N = 15$.

the increase of the refractive index, in view of the Fresnel coefficient only.

The shadow model will now be used in order to gain additional insight to the results presented in Fig. 2. By replacing the rigorous diffraction efficiency η_R with the diffraction efficiency of the "shadow model" η_{SH}, the relative error with respect to the thin element approximation is given (for the case of $n = 1.5$) by

$$\%\text{error} = 100 \cdot \left[4m\frac{N-1}{N}\frac{\lambda}{d}\tan(\theta_{out})\right]. \tag{15}$$

For large ratios of period-length to wavelength one has $\tan(\theta_{out}) \approx \sin(\theta_{out}) = m(\lambda/d)$. Assuming a large number of phase levels (so that $(N-1)/N \to 1$), the relative error can be expressed as

$$\%\text{error} \approx 100 \cdot 4m^2\left(\frac{\lambda}{d}\right)^2. \tag{16}$$

Thus, for a constant error value there is a linear relationship between the grating height and the ratio of period-length to wavelength

$$m \approx \frac{d}{\lambda}\sqrt{\frac{\%\text{error}}{400}}. \tag{17}$$

Inspection of Fig. 2 indeed reveals that the slope increases with the square root of the error.

So far, normal incident angles have been assumed. The shadow concept holds for non-perpendicular angles as well (see Fig. 8), as long as relatively small incident angles are assumed. Three relevant factors should be addressed:

(A) The outgoing angle, θ_{out}, used for computing the shadow effect is now found by using the generalized grating equation

$$d(\sin(\theta_{out}) - n\sin(\theta_{in})) = m\lambda. \tag{18}$$

(B) The TE polarization Fresnel transmission coefficient increases. It now becomes

$$T = \frac{4n\cos(\theta_{in})\sqrt{1-n^2\sin^2(\theta_{in})}}{\left[n\cos(\theta_{in}) + \sqrt{1-n^2\sin^2(\theta_{in})}\right]^2}. \tag{19}$$

(C) For non-perpendicular angles the phase delay increases in view of a longer path within the material, resulting in a decrease of the diffraction efficiency. According to [12], the phase delay is given by

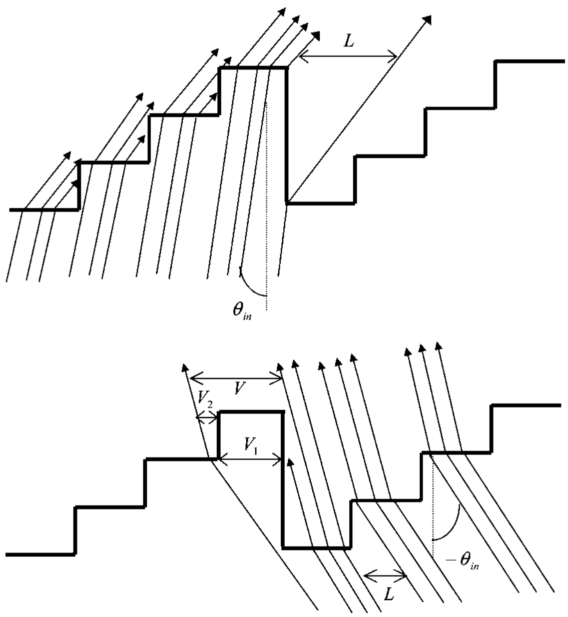

Fig. 8. Non-perpendicular incident angles: top, positive incident angle; bottom, negative incident angle.

$$\phi(\theta) = \frac{2\pi}{\lambda}h\left[n\cos(\theta_{in}) - \sqrt{1-n^2\sin^2(\theta_{in})}\right], \tag{20}$$

where h is the normal depth of the grating. Thus, the relative phase error is

$$r = \frac{\phi(\theta)}{\phi(\theta=0)} = \frac{\left[n\cos(\theta_{in}) - \sqrt{1-n^2\sin^2(\theta_{in})}\right]}{n-1}. \tag{21}$$

The relation between θ_{out} and θ_{in} is provided by Snell's law. This approximation seems to yield reasonable results.

A distinction should be made between negative and positive incident angles, as demonstrated in Fig. 8. While for positive incident angles the shadow terms are still valid (provided that the restrictions mentioned above are taken into account), the case of negative incident angles requires further analysis.

(1) Shadowing between neighboring periods – as shown in Fig. 8, a "critical ray" (i.e. the ray that impinge on the edge of a period) having a negative incident angle will emerge from the grating at a distance V compare with the next ray (going

through the next period), thus creating a "void", reducing the diffraction efficiency. By assuming large number of phase levels (i.e. smooth profile), and by using several simple trigonometric relations, this distance (normalized by the grating period) is found to be

$$\mathrm{SH}_1 = \frac{V}{d} = \frac{N-1}{N(n-1)}\frac{m\lambda}{d}\,\mathrm{tg}(-\theta_{\mathrm{in}}) \times \left[\frac{1-\frac{N-1}{N(n-1)}\frac{m\lambda}{d}\,\mathrm{tg}(\theta_{\mathrm{out}})}{1-\frac{N-1}{N(n-1)}\frac{m\lambda}{d}\,\mathrm{tg}(\theta_{\mathrm{in}})}\right]. \quad (22)$$

(2) Shadowing between neighboring "stairs" – some of the incident rays hit the vertical wall of the phase step, and considered to be lost. The portion of these rays is given by

$$\mathrm{SH}_2 = \frac{N-1}{N}\,\frac{m}{n-1}\,\frac{\lambda}{d}\tan(-\theta_{\mathrm{in}}). \quad (23)$$

Moreover, since the outgoing angle can still be positive (for $m(\lambda/d) > n\sin(\theta_{\mathrm{in}})$), emerging rays may still hit the phase step barrier. As a result, the total shadow resulted in by the staircase structure is now given by

$$\mathrm{SH}_2^{\mathrm{Tot}} = \frac{N-1}{N}\,\frac{m}{n-1}\,\frac{\lambda}{d}\tan(-\theta_{\mathrm{in}}) + \frac{N-1}{N} \times \frac{m}{n-1}\,\frac{\lambda}{d}\tan(\theta_{\mathrm{out}})H(\theta_{\mathrm{out}}), \quad (24)$$

where $H(x)$ is the Heaviside function, defined as

$$H(x) = \begin{cases} 1, & x \geqslant 0, \\ 0, & x \geqslant 0. \end{cases}$$

As can be seen the two shadow factors are no longer identical.

Taking into account these two shadow factors, the expected efficiency is now given by

$$\eta = \frac{4n\cos(\theta_{\mathrm{in}})\sqrt{1-n^2\sin^2(\theta_{\mathrm{in}})}}{\left[n\cos(\theta_{\mathrm{in}})+\sqrt{1-n^2\sin^2(\theta_{\mathrm{in}})}\right]^2}\left[\mathrm{sinc}\left(r\frac{m}{N}\right)\right]^2 \cdot \begin{Bmatrix} [1-2k\tan(\theta_{\mathrm{out}})] & \text{if } \theta_{\mathrm{in}} \geqslant 0, \\ 1-k\tan(\theta_{\mathrm{out}})H(\theta_{\mathrm{out}})-k\tan(-\theta_{\mathrm{in}}) & \\ -k\tan(-\theta_{\mathrm{in}})\left[\frac{1-k\cdot\mathrm{tg}(\theta_{\mathrm{out}})}{1-k\cdot\mathrm{tg}(\theta_{\mathrm{in}})}\right] & \text{if } \theta_{\mathrm{in}} < 0, \end{Bmatrix} \quad (25)$$

where

$$k = \frac{N-1}{N}\,\frac{m}{n-1}\,\frac{\lambda}{d}.$$

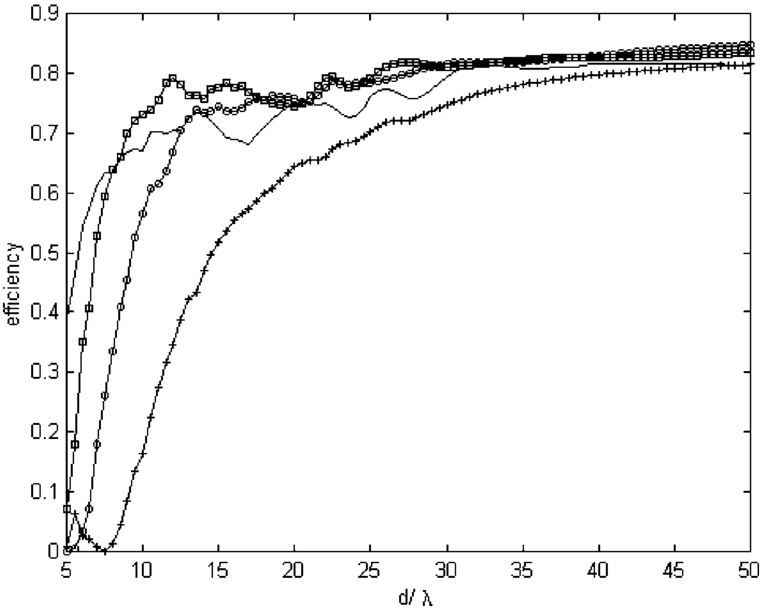

Fig. 9. Efficiency vs. d/λ for several values of incident angles. Solid: −0.2 rad; square: −0.1 rad; circle: 0 rad; plus: +0.1 rad.

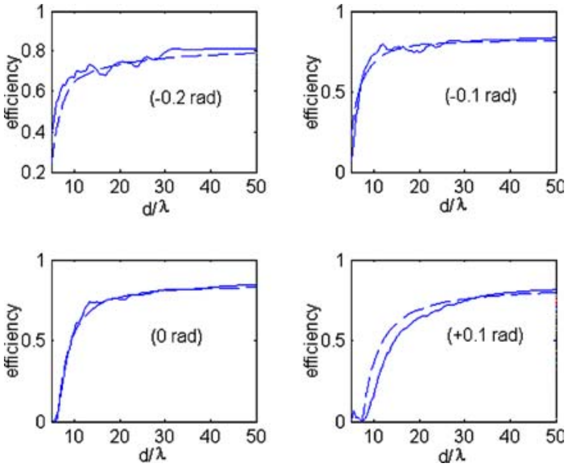

Fig. 10. Efficiency vs. d/λ for several values of incident angles. Solid line: RCWA results; dot-dashed line: shadow model results.

The diffraction efficiency vs. d/λ was calculated using the RCWA approach for several incident angles. Values of $N = 15$, $m = 3$ and $n = 1.5$ were assumed. The results are plotted in Fig. 9.

Fig. 10 compares the results predicted by the shadow model with the accurate results computed according to the RCWA approach. It is clear that the model can track the general efficiency trend, although some deviations can be observed.

4. Model extension – coupling of higher orders

As indicated in the previous section, the accuracy of the shadow model decreases for small number of phase levels. In particular, this is true for the binary equivalent grating ($N/m = 2$) and to some extent for $N/m = 3$, as well. This section provides a physical explanation, and proposes to extend the model so that it can cope better with these highly quantized gratings.

The reason for the deviation of the predicted results can be understood by considering the case of a binary grating. Neglecting Fresnel reflection, the diffraction efficiency predicted by the TEA is ~40%. Due to symmetry one should expect a total of 50% diffraction efficiency for all positive orders (and a similar amount for the negative orders), so that ~10% of the energy should be carried by higher positive diffraction orders. However, for low period to wavelength ratio, some of these orders cannot propagate since $p(\lambda/d) > 1$ (where p is the diffraction order). Moreover, other orders that can propagate will undergo significant shadowing, generally larger in comparison with the shadow of the m's order (which according to TEA model is the lowest order capable of carrying energy). Since energy conservation must be fulfilled, these orders are expected to be coupled into lower orders, raising the total energy diffracted by the m's order.

Following the above discussion, the shadow model can now be refined. We are making a brought force analysis by assuming that the portion of the energy potentially carried by the positive higher diffraction orders but blocked by the shadow effects is coupled into the main diffraction order (the m's order). Using Eq. (2), the anticipated diffraction efficiency is now given by

$$\eta\left(m,N,\frac{\lambda}{d}\right) = \frac{4n}{(n+1)^2}\left\{\left[\operatorname{sinc}\left(\frac{m}{N}\right)\right]^2\left(1-2\frac{N-1}{N}\frac{m}{n-1}\frac{\lambda}{d}\tan(\theta_{\text{out}})\right) + \sum_{q=1}^{\infty}\left[\operatorname{sinc}\left(\frac{p}{N}\right)\right]^2\cdot\max\left[2\frac{N-1}{N}\frac{m}{n-1}\frac{\lambda}{d}\tan(\theta_{\text{out}}),1\right]\right\}, \tag{26}$$

where $q = (p - m)/N$ and θ_{out} is given by the grating equation (Eq. (7)) (m is now replaced by the diffraction order p).

We will now use Eq. (26) to calculate the diffraction efficiency of four different multi-level blazed gratings ($m = 2$, $m = 3$, $N/m = 2$, $N/m = 3$). The results are given in Fig. 11. For comparison purposes, we also present the results obtained by the RCWA approach as well as the prediction of the basic model (Eq. (12)). As can be seen, the refined model that takes into account high orders of diffraction coupled into the main one (dotted curve) predicts the diffraction efficiency with much higher accuracy compared to the basic model (dashed curve), particularly for equivalent binary gratings ($N/m = 2$).

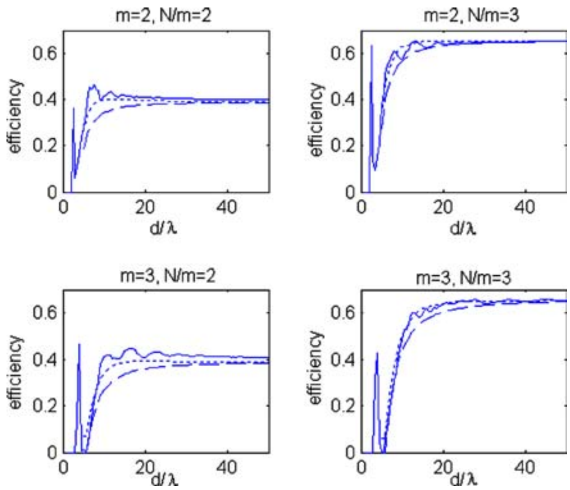

Fig. 11. Efficiency vs. d/λ for highly quantized gratings. Solid line: RCWA results; dashed line: basic shadow model results; dotted line: refined model.

5. Discussion and conclusions

The diffraction efficiency of DOEs is a key parameter in the design of DOE based systems. By using the TEA the efficiency of a periodic DOE can be calculated easily, using a well-known equation. Unfortunately, the TEA is valid only for large period-length to wavelength ratio, and for thin DOEs. Nowadays, many DOEs do not fall into this category, mainly as a result of improved fabrication equipment, and the capabilities of fabricating sub-micron structures, with high aspect ratio.

The diffraction efficiency of a periodic DOE can be found rigorously, by applying a computer algorithm (such as RCWA). However, applying the algorithm require massive computational efforts and large memory, especially when the period-length to wavelength ratio increases.

We compared accurate diffraction efficiency results (obtained by RCWA) with the TEA values, and established an error map, predicting the error caused by using the TEA for various values of period-length to wavelength ratio and phase modulation factor m (represent a maximal phase modulation of $2\pi m$). It turned out that for moderate period to wavelength ratio (order of magnitude of 5–50) the results obtained by the TEA are not accurate. In order to overcome this difficulty, a shadow model was suggested.

The suggested approach is an extension of previously published models, and based on ray optics considerations. The diffraction efficiency predicted by the TEA was corrected by subtracting rays that cannot propagate due to shadows that appear as a result of the staircase structure and of the transition between two adjacent periods. Using the suggested model, an equation predicting the diffraction efficiency as a function of the period-length to wavelength ratio, the phase modulation factor and the number of phase levels with surprisingly high accuracy was developed. The suggested model was also extended to cope with non-perpendicular incident angles and with highly quantized (e.g. binary or equivalent binary) gratings.

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