Efficiency analysis of diffractive lenses

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Received October 15, 1999; revised manuscript received May 30, 2000; accepted July 25, 2000

Multilevel diffractive optical elements are necessary for achieving high-efficiency performance. Here the diffraction efficiency of a multilevel phase-only diffractive lens is analyzed. Approximate, as well as more accurate, approaches are presented. Both plane-wave and Gaussian illumination are discussed. It is shown that for many practical cases the diffraction efficiency can be determined by only a single parameter that takes into account the spatial bandwidth product as well as the focal length of the lens and the illumination wavelength. The analysis is based on the scalar theory and the thin-element approximation. Justification for doing this is presented. The results are valid for lenses with at least $F/5$.

OCIS codes: 050.1380, 050.1970, 220.3630, 220.3740.

1. INTRODUCTION

Diffractive lenses have been well known for many years. The first diffractive lens was the zone plate, suggested and implemented by Lord Rayleigh in an unpublished work from 1871. He showed that a light beam can be focused by using an alternating black and white zone structure, with gradually decreasing period. A very early short summary of zone-plate properties can be found in Ref. 1. Lord Rayleigh also noted that the light intensity obtained at the focus could be multiplied by a factor of 4 if the black areas were replaced by a phase-shifting mask, as also described in Ref. 1. It can be said that this binary-phase zone plate was the first phase-only diffractive element.

Although zone plates were widely used over a long period of time, it took several more decades to fabricate a multilevel phase-only diffractive lens, probably because of the lack of advanced technology, mainly advanced photolithographic processes. However, today multilevel phase-only diffractive lenses are very common.[2,3]

The diffraction efficiency of the diffractive optical element (DOE) is determined mainly by the amount of phase quantization, i.e., by the number of phase levels available. Based on the scalar-theory approximation and the thin-element approximation, the diffraction efficiency of diffractive lenses can approach 100%, provided that a large number of phase levels can be accommodated. It is common to think that the number of phase levels is limited by the depth resolution of the fabrication equipment (or by the number of lithographic masks if a binary-optics[4] fabrication process is assumed). Nevertheless, the space–bandwidth product might also limit the number of phase levels. For example, for a minimal feature size of 5 μm, a 0.5-μm wavelength, and $F/10$, only two phase levels will be available at the edges of the lens. Owing to the above quantization, lens performance will be suppressed significantly. Several authors have tried to face these limitations and improve diffractive-lens performance.[5–9]

Although the diffraction efficiency of diffractive lenses has been analytically analyzed before,[6] the analysis was based on several approximations, which might not reflect the real behavior of the lens.

In this paper a detailed calculation of the diffraction efficiency as a function of the diffractive-lens parameters is given. Approximate as well as more accurate expressions for the diffraction efficiency of diffractive lenses are given. Both a one-dimensional lens (cylindrical lens) and a two-dimensional lens (spherical lens) are discussed. In addition to the conventional plane-wave illumination, the interesting case of Gaussian illumination is also discussed.

Section 2 provides approximate expressions. In Section 3 more accurate results are given. Section 4 compares the two approaches. The comparison is based on several computer simulations. The validity of the thin-element approximation on which the analysis is based is discussed in Section 5. Conclusions are given in Section 6.

2. APPROXIMATE APPROACH

The suggested model assumes that for given working parameters, the scalar theory and the thin-element approximation are valid (for justification see Section 5). In addition, the number of available lithographic masks is assumed to be large enough.

The diffraction efficiency of an N-phase-level DOE is determined according to

$$\eta_N = \left[\frac{\sin(\pi/N)}{\pi/N}\right]^2. \tag{1}$$

The maximal value of N is given by

$$N_{\max} = \frac{\lambda}{(n-1)R}, \tag{2}$$

where λ is the designed illumination wavelength, n is the refractive index, and R is the depth resolution of the fabrication equipment. However, the spatial bandwidth product may also limit the number of available phase levels. Therefore the dependence of the maximal phase levels in the spatial bandwidth product should be developed.

The phase function of a refractive parabolic lens, which is the first approximation of a spherical lens, is given by

$$\phi = \frac{2\pi}{\lambda}[F - (F^2 - r^2)^{1/2}], \tag{3}$$

where F is the focal length of the lens and r is the lateral (or radial) coordinate. The bending angle of the lens (θ), which is measured from the optical axis, can be related to the lens phase function by

$$|k||\sin(\theta)| = \left|\frac{\mathrm{d}\phi}{\mathrm{d}r}\right|, \tag{4}$$

where $|k|$ is the magnitude of the wave vector. Therefore, when Eq. (3) is substituted into Eq. (4), the resulting refractive angle is given by

$$\sin(\theta) = \frac{r}{(F^2 + r^2)^{1/2}} \approx \frac{r}{F}. \tag{5}$$

As will be explained below, the analysis is valid for lenses with F-number of $F/5$ or more. Thus $r \leqslant 0.1F$, and the error caused by using the above approximation is marginal. The first-order diffractive angle is given by the grating formula

$$d \sin(\theta) = \lambda, \tag{6}$$

where d is the grating period. Therefore the period of the diffractive lens can be calculated by equating the diffractive angle and the refractive angle:

$$d = \frac{\lambda F}{r}. \tag{7}$$

As can be seen, the period becomes smaller for larger values of r. Therefore the number of phase levels might also be affected by the limited spatial bandwidth product. If we define a new parameter B—the minimal feature size—the local maximum number of phase levels N_{mL} is given by

$$N_{mL} = \frac{d}{B} = \frac{\lambda F}{rB}. \tag{8}$$

For the case of high depth resolution we can assume that the diffraction efficiency is controlled by the lateral resolution only. Therefore the local efficiency of the lens is given by

$$\eta_L(r) = \left[\frac{\sin(\pi/N_{mL})}{\pi/N_{mL}}\right]^2 = \left[\frac{\sin(\pi Br/\lambda F)}{\pi Br/\lambda F}\right]^2. \tag{9}$$

Assuming plane-wave illumination, the diffraction efficiency can be approximated by integrating with respect to the lateral coordinate,

$$\bar{\eta} = \frac{2}{D}\int_0^{D/2} \eta_L(r)\mathrm{d}r = \frac{2}{D}\int_0^{D/2}\left[\frac{\sin(\pi Br/\lambda F)}{\pi Br/\lambda F}\right]^2 \mathrm{d}r \tag{10}$$

for a cylindrical lens and

$$\bar{\eta} = \frac{8}{D^2}\int_0^{D/2} r\,\eta_L(r)\mathrm{d}r = \frac{8}{D^2}\int_0^{D/2} r\left[\frac{\sin(\pi Br/\lambda F)}{\pi Br/\lambda F}\right]^2 \mathrm{d}r \tag{11}$$

for a spherical lens, where D is the lens diameter. Similar expressions are also given in Ref. 6.

The solution of these expressions is given by

$$\bar{\eta} = 2\left(\frac{\lambda F}{\pi BD}\right)^2\left[\cos\left(\frac{\pi BD}{\lambda F}\right) + Si\left(\frac{\pi BD}{\lambda F}\right)\frac{\pi BD}{\lambda F} - 1\right] \tag{12}$$

for a cylindrical lens and

$$\bar{\eta} = \left(\frac{2\lambda F}{\pi BD}\right)^2\left[\text{eulergamma} - \log\left(\frac{\lambda F}{\pi BD}\right) - Ci\left(\frac{\pi BD}{\lambda F}\right)\right] \tag{13}$$

for a spherical lens. Where the constant eulergamma $= 0.57721$,

$$Si(x) = \int_0^x \frac{\sin(t)}{t}\mathrm{d}t$$

and

$$Ci(x) = \text{eulergamma} + \log(x) + \int_0^x \frac{\cos(t) - 1}{t}\mathrm{d}t.$$

We now define a quality parameter $Q = (BD)/(\lambda F)$ so that these expressions can be simplified:

$$\bar{\eta} = \frac{2}{\pi Q^2}[\cos(\pi Q) + Si(\pi Q)\pi Q - 1] \tag{14}$$

$$\bar{\eta} = \left(\frac{2}{\pi Q}\right)^2\left[\text{eulergamma} - \log\left(\frac{1}{\pi Q}\right) - Ci(\pi Q)\right]. \tag{15}$$

For $0 < Q < 1$, Eqs. (14) and (15) can be replaced by the following polynomials:

$$\bar{\eta} \cong 1 - 0.27Q^2 + 0.045Q^4, \tag{16}$$

$$\bar{\eta} \cong 1 - 0.4Q^2 + 0.068Q_4 \tag{17}$$

for the cylindrical and the spherical cases, respectively. The error caused by using the polynomial approximation is negligible.

3. MORE ACCURATE CALCULATION

The expressions given in Section 2 for the efficiency of the diffractive lens are not completely accurate, for three main reasons:

1. The scalar theory and the thin-element approximation were assumed to be valid.
2. Limited depth was ignored, and this decision needs justification.
3. Continuous rather than discrete spatial dependence of the phase levels was assumed [integration rather than summation in Eq. (10)].

In the following, a more accurate calculation is performed. Although scalar theory and the thin-element approximation are still used, the other two approximations are now removed.

As discussed above, the available number of phase levels is limited either by the number of masks or by the lateral resolution. As can be seen from Eq. (7), the period size of the lens becomes finer toward the edges. Thus the diffractive lens contains regions with different numbers of phase levels. Therefore the efficiency of the lens should be calculated as a weighting sum of the contributions from each of these regions:

$$\bar{\eta} = \sum_{N_L=N_{\min}}^{N_{\max}} \gamma_{N_L}\eta_{N_L}, \tag{18}$$

where $N_{\min}$ is the minimal number of available phase levels, $N_{\max}$ is the maximal number of phase levels [given by Eq. (2)], γ_{N_L} is the weighting coefficient of each N_L phase level that exists, and η_{N_L} is the efficiency of that region [per Eq. (1)].

Since N, the number of phase levels, is an integer, $N_{\min}$ is determined by the minimal feature size available, according to

$$N_{\min} = \left\{\begin{array}{ll} \mathrm{fix}\left(\dfrac{2\lambda F}{BD}\right) = \mathrm{fix}(2/Q) & \text{if } 2/Q < N_{\max} \\ N_{\max} & \text{otherwise} \end{array}\right\}, \tag{19}$$

where $Q = (BD)/(\lambda F)$ is the newly defined quality factor and fix(x) rounds the value of x to the nearest lower integer.

Assuming plane-wave illumination, the contribution of each N_L is given by the relative area of the lens, which has N_L phase levels. Therefore γ_{N_L} can be calculated according to

$$\gamma_{N_L} = \int_{r_{\min}}^{r_{\max}} \mathrm{d}r \Big/ \int_0^{D/2} \mathrm{d}r = \frac{2}{D}(r_{\max} - r_{\min}) \tag{20}$$

for cylindrical lenses and

$$\gamma_{N_L} = \int_{r_{\min}}^{r_{\max}} r\mathrm{d}r \Big/ \int_0^{D/2} r\mathrm{d}r = \frac{4}{D^2}(r_{\max}^2 - r_{\min}^2) \tag{21}$$

for spherical lenses, where $r_{\max}$ and $r_{\min}$ bound the region in which N_L phase levels exist.

Using Eq. (20), which relates the lateral location r to the period d, we developed the following algorithm for calculation of the boundaries in Eqs. (20) and (21):

$$r_{\min} = \begin{array}{ll} 0,\ r_{\max} = D/2,\ N_{\min} = N_{\max} & \text{if } N_L = N_{\max} \leqslant 2/Q \\ \dfrac{\lambda F}{B(N_L+1)},\ r_{\max} = \dfrac{\lambda F}{BN_L} & \text{if } 2/Q < N_L < N_{\max} \\ \dfrac{\lambda F}{B(N_L+1)},\ r_{\max} = D/2 & \text{if } N_L \leqslant 2/Q \\ 0,\ r_{\max} = \dfrac{\lambda F}{BN_L} & \text{otherwise (if } N_L = N_{\max} > 2/Q) \end{array}. \tag{22}$$

By substituting the results of Eq. (22) into Eqs. (20) and (21), one obtains the value for γ_{N_L}:

$$\gamma_{N_L} = \begin{cases} 1 & \text{if } N_L = N_{\max} \leqslant 2/Q \\ (2/Q)\dfrac{1}{N_L(N_L+1)} & \text{if } 2/Q < N_L < N_{\max} \\ 1 - \dfrac{1}{N_L+1}(2/Q) & \text{if } N_L \leqslant 2/Q \\ \dfrac{1}{N_L}(2/Q) & \text{otherwise (if } N_L = N_{\max} > 2/Q) \end{cases} \tag{23}$$

for a cylindrical lens and

$$\gamma_{N_L} = \begin{cases} 1 & \text{if } N_L = N_{\max} \leqslant 2/Q \\ (2/Q)^2\dfrac{2N_L+1}{[N_L(N_L+1)]^2} & \text{if } 2/Q < N_L < N_{\max} \\ 1 - \dfrac{1}{(N_L+1)^2}(2/Q)^2 & \text{if } N_L \leqslant 2/Q \\ \dfrac{1}{N_L^2}(2/Q)^2 & \text{otherwise (if } N_L = N_{\max} > 2/Q) \end{cases} \tag{24}$$

for a spherical lens.

Up to now, only plane-wave illumination was assumed. However, the case of Gaussian illumination is more relevant for many optical systems. In order to use the obtained results for Gaussian illumination, we should replace Eqs. (20) and (21) with

$$\gamma_{N_L} = \frac{\int_{r_{\min}}^{r_{\max}} \exp[-2(r/w)^2]\mathrm{d}r}{\int_0^{D/2} \exp[-2(r/w)^2]\mathrm{d}r} = \frac{\operatorname{erf}(r_{\max}\sqrt{2}/w) - \operatorname{erf}(r_{\min}\sqrt{2}/w)}{\operatorname{erf}(D/w\sqrt{2})}, \tag{25}$$

$$\gamma_{N_L} = \frac{\int_{r_{\min}}^{r_{\max}} r\exp[-2(r/w)^2]\mathrm{d}r}{\int_0^{D/2} r\exp[-2(r/w)^2]\mathrm{d}r} = \frac{\exp[-2(r_{\min}/w)^2] - \exp[-2(r_{\max}/w)^2]}{1 - \exp\left[-\frac{1}{2}(D/w)^2\right]}, \tag{26}$$

respectively.

With Eq. (22) the obtained weighting coefficients are

$$\gamma_{N_L} = \begin{cases} 1 & \text{if } N_L = N_{\max} \leqslant 2/Q \\[2ex] \dfrac{\operatorname{erf}\left(\dfrac{\sqrt{2}}{Q}\dfrac{D}{w}\dfrac{1}{N_L}\right) - \operatorname{erf}\left(\dfrac{\sqrt{2}}{Q}\dfrac{D}{w}\dfrac{1}{N_L+1}\right)}{\operatorname{erf}\left(\dfrac{D}{w}\dfrac{1}{\sqrt{2}}\right)} & \text{if } 2/Q < N_L < N_{\max} \\[2ex] 1 - \dfrac{\operatorname{erf}\left(\dfrac{\sqrt{2}}{Q}\dfrac{D}{w}\dfrac{1}{N_L+1}\right)}{\operatorname{erf}\left(\dfrac{D}{w}\dfrac{1}{\sqrt{2}}\right)} & \text{if } N_L \leqslant 2/Q \\[2ex] \dfrac{\operatorname{erf}\left(\dfrac{\sqrt{2}}{Q}\dfrac{D}{w}\dfrac{1}{N_L}\right)}{\operatorname{erf}\left(\dfrac{D}{w}\dfrac{1}{\sqrt{2}}\right)} & \text{otherwise (if } N_L = N_{\max} > 2/Q) \end{cases}, \tag{27}$$

$$\gamma_{N_L} = \begin{cases} 1 & \text{if } N_L = N_{\max} \leqslant 2/Q \\[2ex] \dfrac{\exp\left[-2\left(\dfrac{1}{Q}\dfrac{D}{w}\dfrac{1}{N_L+1}\right)^2\right] - \exp\left[-2\left(\dfrac{1}{Q}\dfrac{D}{w}\dfrac{1}{N_L}\right)^2\right]}{1 - \exp\left[-\dfrac{1}{2}\left(\dfrac{D}{w}\right)^2\right]} & \text{if } 2/Q < N_L < N_{\max} \\[2ex] \dfrac{\exp\left[-2\left(\dfrac{1}{Q}\dfrac{D}{w}\dfrac{1}{N_L+1}\right)^2\right] - \exp\left[-\dfrac{1}{2}\left(\dfrac{D}{w}\right)^2\right]}{1 - \exp\left[-\dfrac{1}{2}\left(\dfrac{D}{w}\right)^2\right]} & \text{if } N_L \leqslant 2/Q \\[2ex] \dfrac{1 - \exp\left[-2\left(\dfrac{1}{Q}\dfrac{D}{w}\dfrac{1}{N_L}\right)^2\right]}{1 - \exp\left[-\left(\dfrac{D}{w}\right)^2\right]} & \text{otherwise (if } N_L = N_{\max} > 2/Q) \end{cases}. \tag{28}$$

It can be noticed from Eqs. (27) and (28) that the behavior of the diffractive lens is now determined by the ratio D/w, in addition to Q. It can be shown [by using the Taylor first-order expansion $\mathrm{erf}(x) \approx x$ and $\exp(x) \approx 1 + x$] that in the limit of $w \gg D$, those equations become the plane-wave equations. On the other hand, for the case of $w \ll D$, $\gamma_{N_{\max}} \rightarrow 1$, so that maximal efficiency is obtained. This is of no surprise, since only a small portion of the energy reaches the edges of the lens, while most of it passes through the center where the number of phase levels is highest.

4. COMPUTER SIMULATIONS

Several computer simulations have been carried out to calculate the efficiency of the diffractive lens and to compare the approximated and the more accurate approaches. Figures 1 and 2 show the diffraction efficiency versus Q for a cylindrical and a spherical lens, respectively, based on the approximate approach. The diffraction efficiency of cylindrical and spherical lenses as a function of Q and $N_{\max}$ for the case of plane-wave illumination is shown. The results of the more accurate model (per Section 3) can be seen in Figs. 3 and 4, respectively. One can notice that the efficiency obtained is lower than that obtained with the approximate approach. This is due to both the limited value of $N_{\max}$ and the use of summation rather than the integral. In addition, for low Q as well as low $N_{\max}$ values, a plateau is reached, because although the lateral resolution is high enough to allow more phase levels, these are unobtainable because we have reached the assumed $N_{\max}$ limit.

The case of Gaussian illumination is shown in Figs. 5 and 6, which depict the efficiency of cylindrical and spherical lenses versus Q on the basis of the more accurate approach for four $N_{\max}$ selections: 4, 8, 16, and 32.

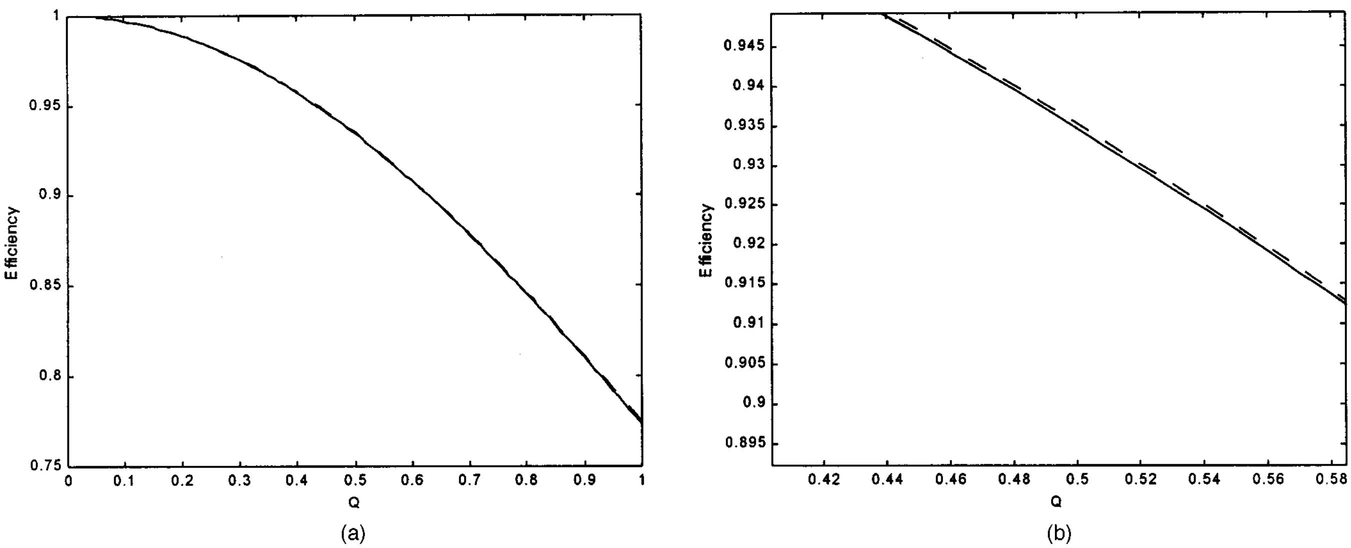

Fig. 1. Efficiency versus Q based on the approximated approach for a cylindrical lens. Solid curves, complete term; dashed curves, polynomial approximation. (a) Full scale, (b) zoom.

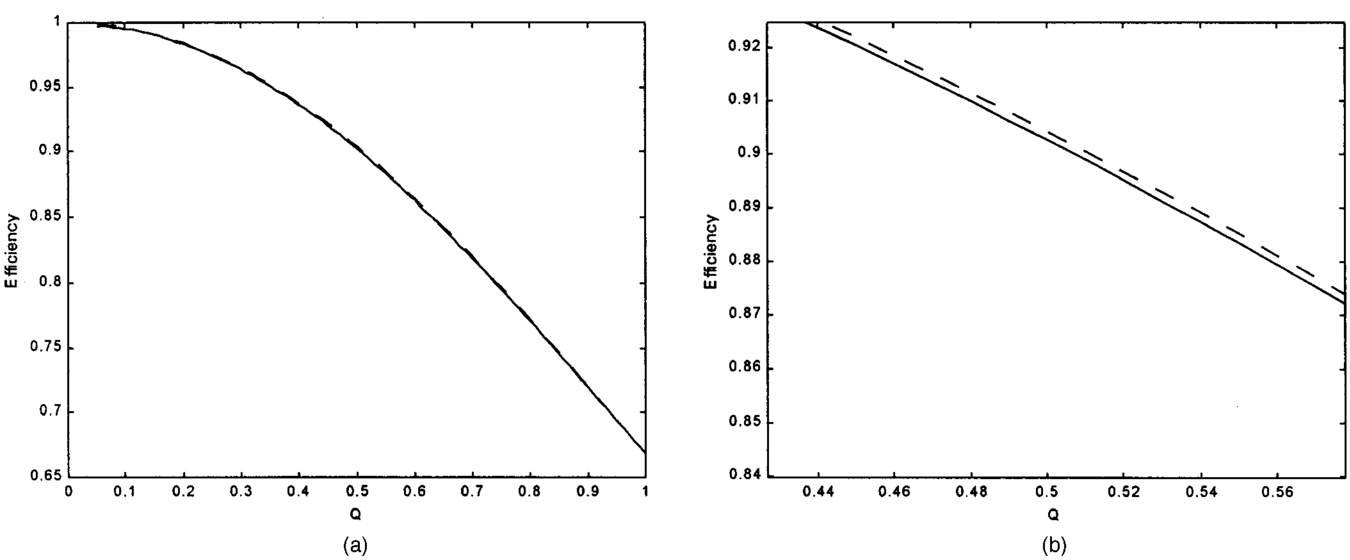

Fig. 2. Efficiency versus Q based on the approximated approach for a spherical lens. Solid curves, complete term; dashed curves, polynomial approximation. (a) Full scale, (b) zoom.

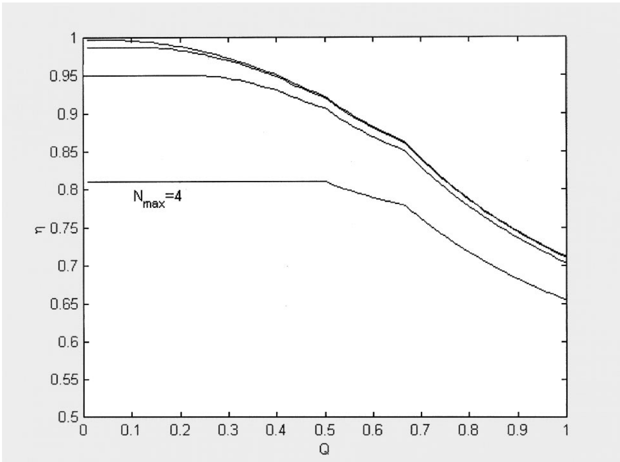

Fig. 3. Efficiency versus Q based on the accurate approach for a cylindrical lens ($N_{\max} = 4, 8, 16, 32$).

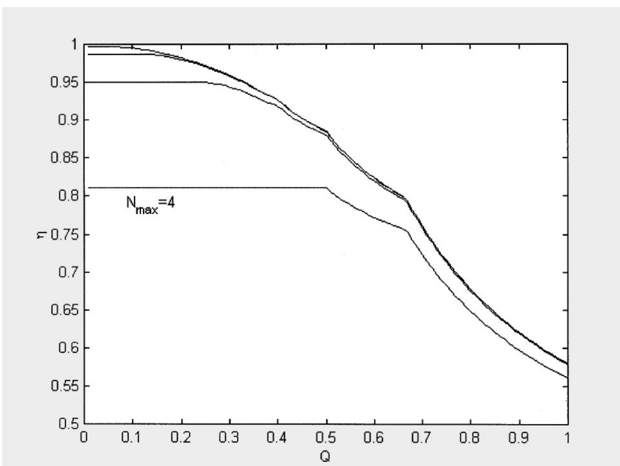

Fig. 4. Efficiency versus Q based on the accurate approach for a spherical lens ($N_{\max} = 4, 8, 16, 32$).

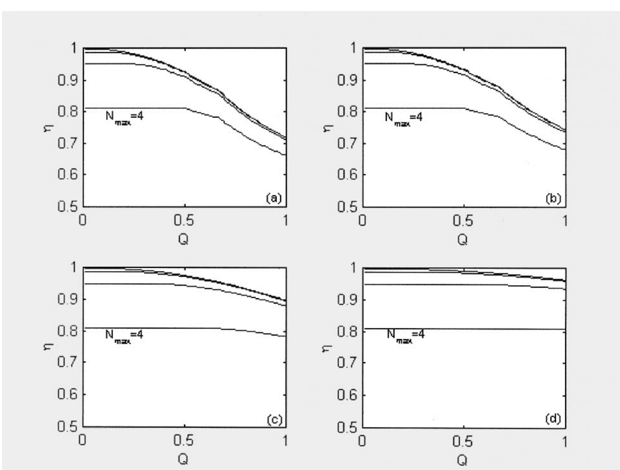

Fig. 5. Efficiency versus Q for Gaussian illumination based on the accurate approach for a cylindrical lens ($N_{\max} = 4, 8, 16, 32$). (a) $D/w = 0.5$, (b) $D/w = 1$, (c) $D/w = 3$, (d) $D/w = 5$.

Several D/w values were examined: (a) 0.5, (b) 1, (c) 3, and (d) 5. It is evident that for high D/w values the efficiency improves significantly with respect to the case of plane-wave illumination because of the relatively larger contribution of the central zone of the lens, in which more phase levels are available. On the other hand, for low D/w values no significant difference can be seen, as expected.

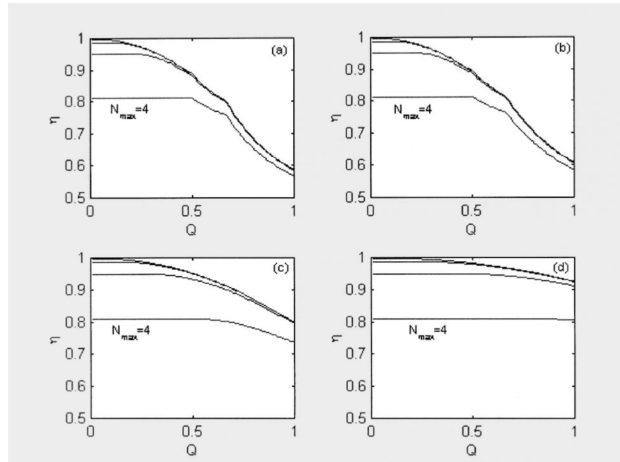

Fig. 6. Efficiency versus Q for Gaussian illumination based on the accurate approach for a spherical lens ($N_{\max} = 4, 8, 16, 32$). (a) $D/w = 0.5$, (b) $D/w = 1$, (c) $D/w = 3$, (d) $D/w = 5$.

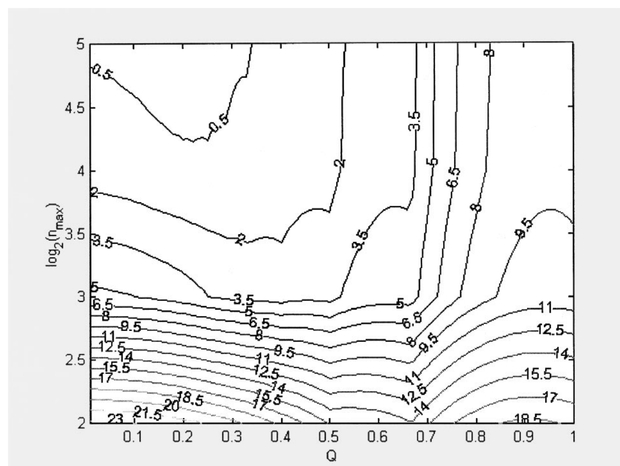

Fig. 7. Error caused by using the approximate approach versus Q and $N_{\max}$ for a cylindrical lens.

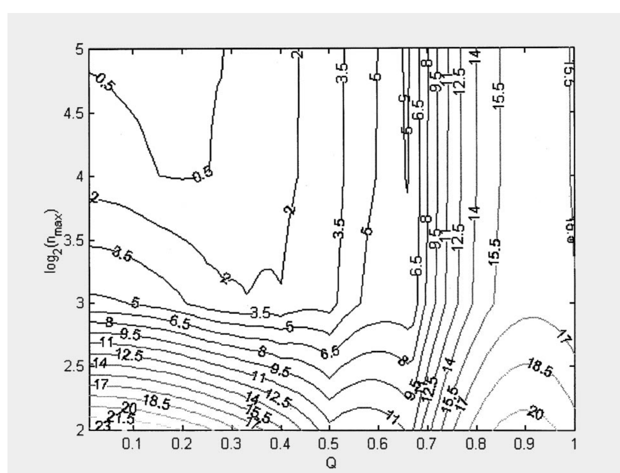

Fig. 8. Error caused by using the approximate approach versus Q and $N_{\max}$ for a spherical lens.

To determine the regions (in the parameter space) in which the approximate approach can be used, two error maps are presented: the error as a function of Q and $N_{\max}$ in Figs. 7 and 8, respectively. The error is defined as the relative difference between the approximate and the more accurate approach, according to

$$\text{error} = \frac{|\eta_{\text{accurate}} - \eta_{\text{approx}}|}{\eta_{\text{accurate}}} * 100. \tag{29}$$

Those figures can serve as a guideline for proper calculation of the diffraction efficiency. The calculation of the diffraction efficiency with the approximate approach is straightforward, whereas the computer algorithm should be applied for use of the more accurate approach. Therefore it is much more convenient to use the approximate solution, and thus the importance of the above figures is clear. It can be seen that for both types of lenses, the obtained error is low for large values of $N_{\max}$ (at least 16) and low Q values (less than 0.4). On the other hand, for $N_{\max} = 4$ (or less), the approximations deviate significantly from the accurate results. Significant deviations occur also for Q values larger than 0.7. However, at present, high values of $N_{\max}$ can be easily achieved, and many practical lenses can be characterized by a Q value lower than 0.4. Therefore for many practical cases the approximate approach can be safely used.

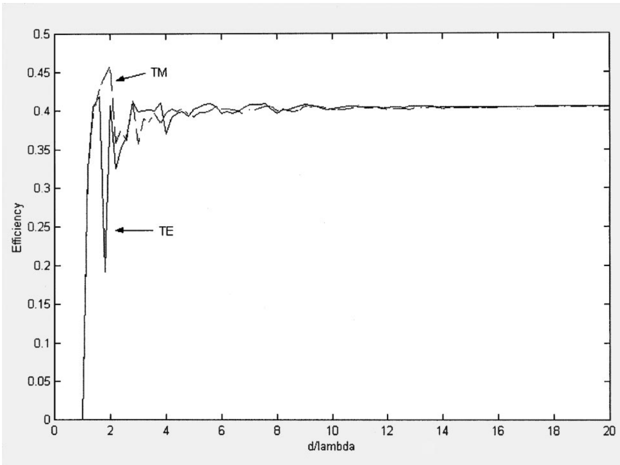

Fig. 9. Diffraction efficiency of a binary grating versus d/λ.

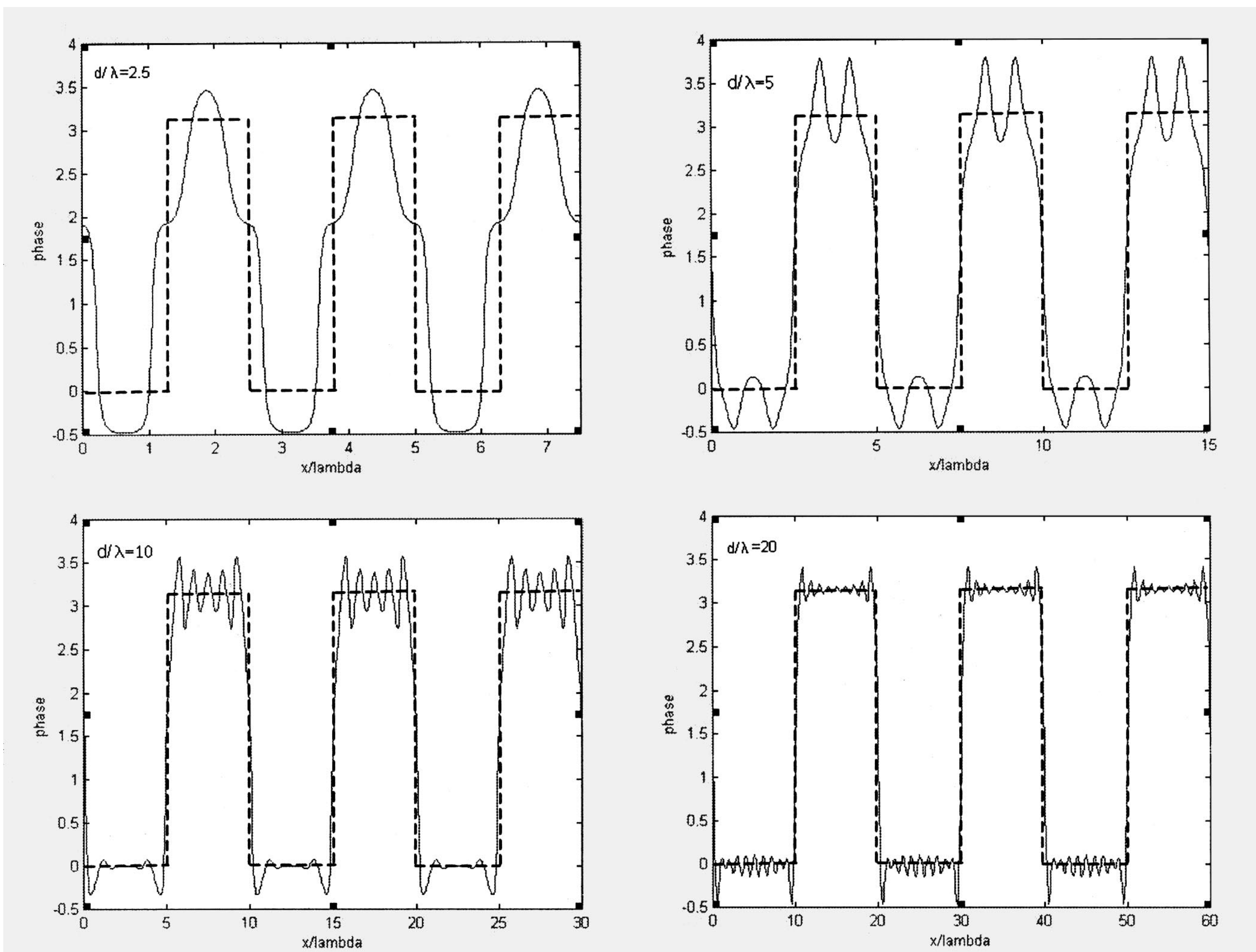

Fig. 10. TE phase front of the outgoing wave for several values of d/λ. Solid curves, real phase profile; dashed curves, desired binary phase profile.

5. VALIDITY OF THE THIN-ELEMENT APPROXIMATION

Throughout the mathematical development in this paper, both the scalar theory and the thin-element approximation have been used. The thin-element approximation assumes that the field beyond the DOE is obtained by multiplication of the incident field and the phase function of the DOE. Such an assumption is valid only for structures with lateral dimensions much larger than the wavelength. How much larger should this be?

To define the region in which the thin-element approximation is still valid, we calculated the field just beyond a binary grating, using rigorous coupled-wave analysis[10,11] (RCWA) as a function of d/λ, and we evaluated the first-order diffraction efficiency (neglecting Fresnel reflections). The results shown in Fig. 9 are very similar to those given by Noponen *et al.*[12] It can be seen that for $d/\lambda > 6$ the efficiency converges to the value predicted by the scalar theory and the thin-element approximation. For a qualitative measure, it may be instructive to evaluate the error caused by using the thin-element approximation by observing the phase front of the outgoing wave calculated by the RCWA approach and to compare it with the square-wave phase front predicted by the thin-element approximation. The TE phase front of the outgoing wave calculated by the RCWA approach for several d/λ values is given in Fig. 10. It can be seen that besides Gibbs phenomena (due to the finite number of harmonics) the phase front is almost a square wave for $d/\lambda = 10$ and $d/\lambda = 20$. Indeed, even for $d/\lambda = 5$ the phase front is not far from the desired one. It can be said that that the thin-element approximation can be safely used for cases in which $d/\lambda \geqslant 10$. For the case of a large number of phase levels, a higher ratio is recommended. However, for the case of $d/\lambda \approx 10$, a large number of phase levels can be obtained only by using very small feature size. By limiting our analysis to minimal feature size of 2.5λ (~ 1.5 μm for the visible region) no more than four phase levels can be used at the edge of the lens. As can be seen also in Ref. 12 the case of four phase levels can be treated by the thin-element approximation, assuming $d/\lambda \geqslant 10$.

As can be seen from Eq. (7), the period at the edge of the lens is given by $d = 2[(\lambda F)/D]$. Therefore $F/D = (1/2)(d/\lambda) \geqslant 5$; i.e., the analysis is valid for lenses of or $F/5$ or higher.

6. CONCLUSIONS

A detailed analysis of the diffraction efficiency of a diffractive lens has been carried out. Approximations as well as more accurate results were given. Plane-wave and Gaussian illumination were discussed. For the plane-wave-illumination case, the error of using the approximations as a function of the lens parameters was calculated and was presented by error maps. The region (in the parameter space) in which the approximate approach is still valid can easily be found by using these maps. It was shown that for many practical cases, a lens can be described by one parameter only, the quality parameter Q, found to be equal to $BD/\lambda F$ or $B/(\lambda F\#)$. The above results are valid for lenses with at least $F5$, with minimal feature size of at least 2.5λ.

ACKNOWLEDGMENTS

The authors thank the Ministry of Science and the Arts for its support of this research in the framework of the National Infrastructure Program. Uriel Levy thanks the Ministry of Science and the Arts for an Eshkol fellowship.

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