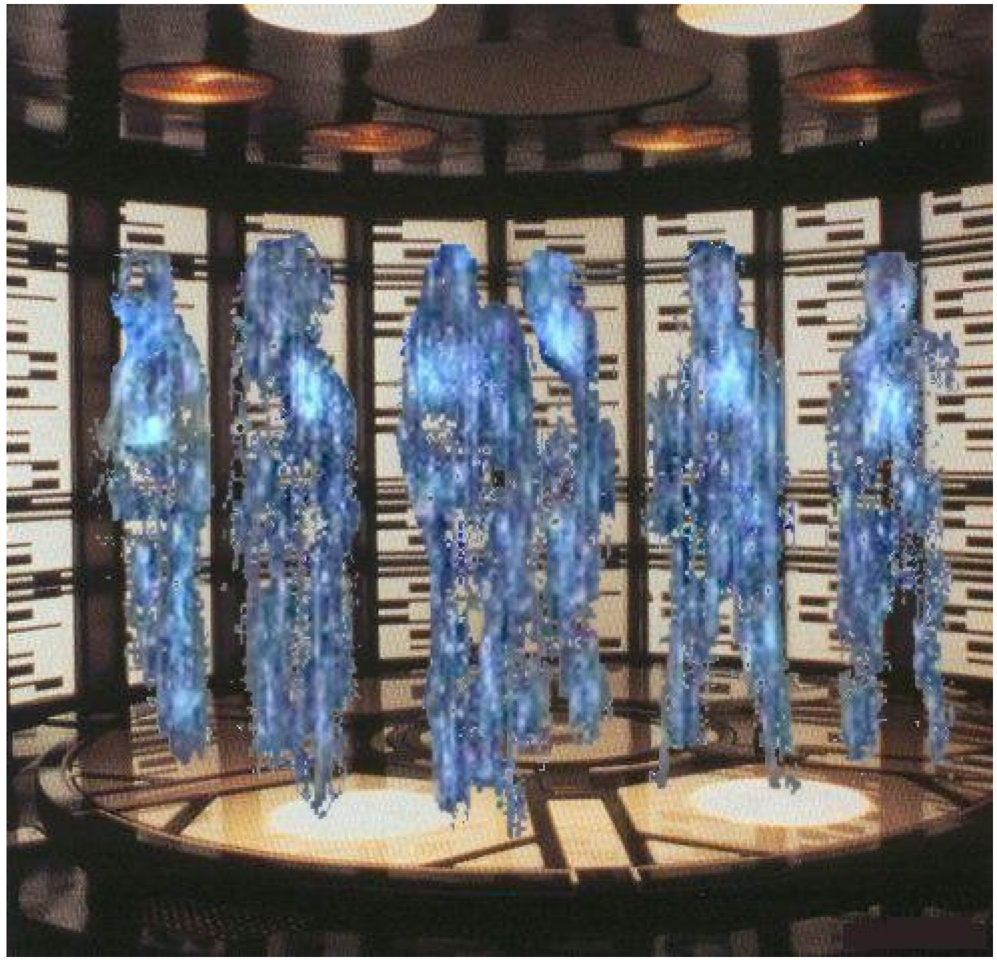
# Quantum Teleportation

$\langle\Psi|$  $|\Phi\rangle$

**Gur Yaari for HEisenberg's Seminar on Quantum Optics**

# Bell States

## Maximum Entangled Quantum States:

**The usual form of the CHSH inequality is:**

$$|E(a, b) - E(a, b') + E(a', b) + E(a' b')| \leq 2$$

**In maximum entangled quantum states:**

$$|E(a, b) - E(a, b') + E(a', b) + E(a' b')| = 2^{2\sqrt{2}}$$

# Bell States

**maximum entangled quantum states of two "qubits"**

**a qubit:** $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$

$$|\Phi^{\pm}\rangle = 1/\sqrt{2}(|0\rangle_A \times |0\rangle_B \pm |1\rangle_A \times |1\rangle_B)$$

$$|\Psi^{\pm}\rangle = 1/\sqrt{2}(|1\rangle_A \times |0\rangle_B \pm |0\rangle_A \times |1\rangle_B)$$

**=> orthonormal basis**

# Bell States

## measurement:

-It is a joint quantum-mechanical measurement of two qubits that determines in which of the four Bell states the two qubits are in.

-If the qubits were not in a Bell state before, they get projected into a Bell state , and as Bell states are entangled, <u>a Bell measurement is an entangling operation.</u>

-In linear optics only partial information can get out of such measurement:

# Bell States

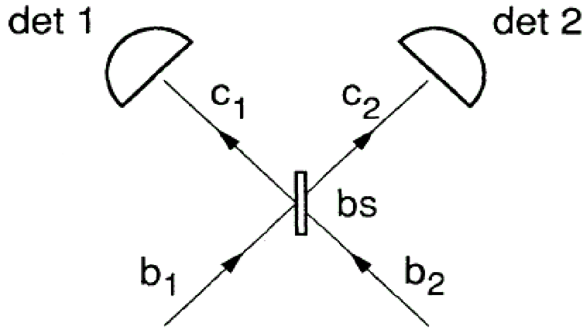
## measurement:

**Samuel L. Braunstein and A. Mann Measurement of the Bell operator and quantum teleportation Phys. Rev. A 51, R1727 - R1730 (1995)**

Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel

## measuring $|\Psi^{\pm}\rangle, |\Phi\rangle$: $|\Psi^{-}\rangle$




$$\hat{c}_1^j = 1/\sqrt{2}(\hat{b}_1^j + \hat{b}_2^j), \quad \hat{c}_2^j = 1/\sqrt{2}(\hat{b}_1^j - \hat{b}_2^j),$$

$$\hat{c}_1^j \, \hat{c}_2^k = \frac{1}{2}(\hat{b}_1^j \, \hat{b}_1^k - \hat{b}_1^j \, \hat{b}_2^k + \hat{b}_2^j \, \hat{b}_1^k - \hat{b}_2^j \, \hat{b}_2^k).$$

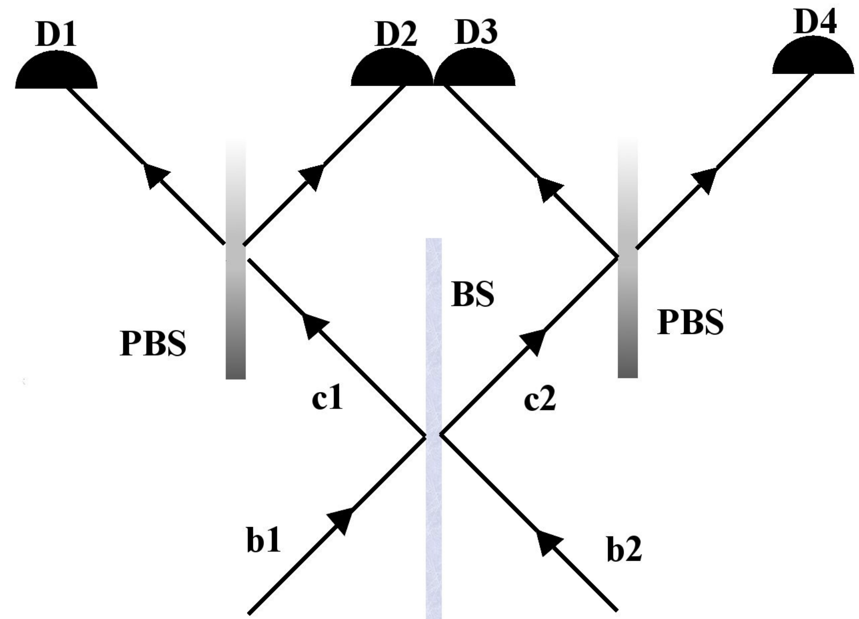
$$\langle 0 | \hat{c}_1^j \hat{c}_2^k = \begin{cases} \pm (1/\sqrt{2}) \, {}_{12}\langle \Psi^{(-)} |, & j \neq k \\ 0, & j = k \end{cases}$$

# Bell States

## measurement:

## measuring $|\Psi^{\pm}\rangle, |\Phi\rangle$: $|\Psi^{+}\rangle$

$$\langle 0|\hat{c}_1^j \hat{c}_1^k = \begin{cases} (1/\sqrt{2})\ {}_{12}\langle\Psi^{(+)}|, & j \neq k \\ (1/\sqrt{2})({}_{12}\langle\Phi^{(+)}| + {}_{12}\langle\Phi^{(-)}|), & j = k = \updownarrow \\ (1/\sqrt{2})({}_{12}\langle\Phi^{(+)}| - {}_{12}\langle\Phi^{(-)}|), & j = k = \leftrightarrow \end{cases}$$



# Bell States

## measurement:

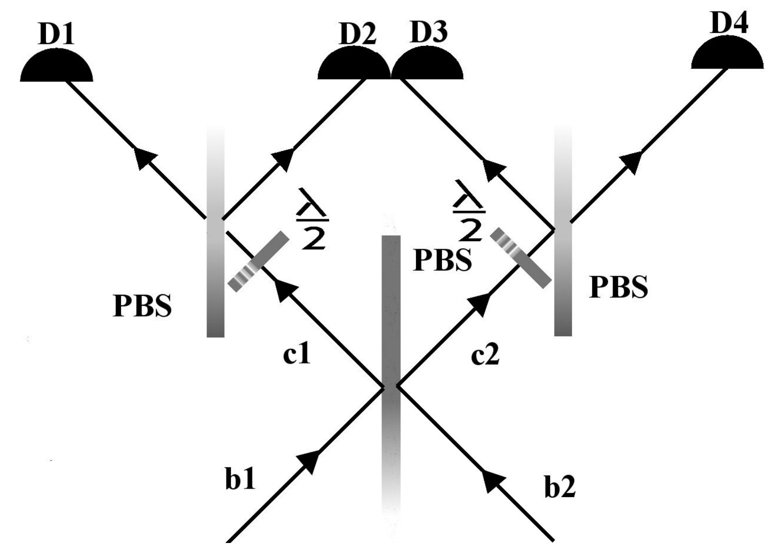
## measuring $|\Phi^{\pm}>, |\Psi>$:

$b1_h \dashrightarrow c2_h - c2_v \Longrightarrow |\Phi^{+}> = b1_h b2_h + b1_v b2_v \longrightarrow c1_h c2_h + c1_v c2_v$

$b1_v \dashrightarrow c1_h + c1_v \Longrightarrow |\Phi^{-}> = b1_h b2_h - b1_v b2_v \longrightarrow c1_h c2_v + c1_v c2_h$

$b2_v \dashrightarrow c2_h + c2_v \Longrightarrow |\Psi^{\pm}> \longrightarrow c1_h c1_v + c1_h c1_h + c1_v c1_v + c2_h c2_h +$

$b2_h \dashrightarrow c1_h - c1_v \qquad\qquad + c2_h c2_v + c2_v c2_v$



# Bell States

**With linear optics one cannot perform a complete measurement of the Bell states.**

$$|\Psi_-\rangle = (1/\sqrt{2})(|\uparrow\rangle_L|\downarrow\rangle_R - |\downarrow\rangle_L|\uparrow\rangle_R)$$

$$|\uparrow\rangle_L \rightarrow \sum a_i|i\rangle, \quad |\downarrow\rangle_L \rightarrow \sum b_i|i\rangle,$$

$$|\uparrow\rangle_R \rightarrow \sum c_i|i\rangle, \quad |\downarrow\rangle_R \rightarrow \sum d_i|i\rangle,$$

**Linearity implies:**

$$|\Psi_-\rangle \rightarrow \sum_{i,j} \alpha_{ij}|i\rangle|j\rangle, \quad |\Psi_+\rangle \rightarrow \sum_{i,j} \beta_{ij}|i\rangle|j\rangle,$$

$$|\Phi_-\rangle \rightarrow \sum_{i,j} \gamma_{ij}|i\rangle|j\rangle, \quad |\Phi_+\rangle \rightarrow \sum_{i,j} \delta_{ij}|i\rangle|j\rangle.$$

$|i\rangle|j\rangle$ signifies properly symmetrized states $(1/\sqrt{2})(|i\rangle_1|j\rangle_2 \pm |j\rangle_1|i\rangle_2)$.

# Bell States

erator means that there is at least one nonzero coefficient of every kind $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij}$ and if, for a certain $i, j$, it is not zero, then all others are zero.

For $i=j$ we have

$$
\begin{aligned}
\alpha_{ii} &= a_i d_i - b_i c_i, \\
\beta_{ii} &= a_i d_i + b_i c_i, \\
\gamma_{ii} &= a_i c_i - b_i d_i, \\
\delta_{ii} &= a_i c_i + b_i d_i
\end{aligned}
\tag{6}
$$

For $i \neq j$ we have

$$
\begin{aligned}
\alpha_{ij} &= a_i d_j + a_j d_i - (b_i c_j + b_j c_i) \\
\beta_{ij} &= a_i d_j + a_j d_i + b_i c_j + b_j c_i \\
\gamma_{ij} &= a_i c_j + a_j c_i - (b_i d_j + b_j d_i) \\
\delta_{ij} &= a_i c_j + a_j c_i + b_i d_j + b_j d_i .
\end{aligned}
$$

immediately that Eq. (7) holds. The measurability of the nondegenerate Bell operator requires that for any given $i$ at least three out of the coefficients $\alpha_{ii}, \beta_{ii}, \gamma_{ii}, \delta_{ii}$ are zero. From Eqs. (6) it follows that the fourth coefficient must be zero too and therefore we obtain Eq. (7) also for the identical bosons. Thus, using Eq. (6) again, we obtain

$$a_i d_i = b_i c_i = a_i c_i = b_i d_i = 0. \tag{10}$$

Therefore at least two coefficients out of four are zero: either $a_i = b_i = 0$ or $c_i = d_i = 0$.

Let us assume now $\alpha_{ij} \neq 0$ (and therefore $\beta_{ij} = \gamma_{ij} = \delta_{ij} = 0$) and assume that $a_i = b_i = 0$. Then, Eqs. (8) become

$$\alpha_{ii} = \beta_{ii} = \gamma_{ii} = \delta_{ii} = 0. \quad (7$$

$$\begin{aligned} \alpha_{ij} &= a_j d_i - b_j c_i \neq 0, \\ \beta_{ij} &= a_j d_i + b_j c_i = 0, \\ \gamma_{ij} &= a_j c_i - b_j d_i = 0, \\ \delta_{ij} &= a_j c_i + b_j d_i = 0. \end{aligned} \tag{11}$$

These equations, however, do not have a solution. It is easy to see that also for all other cases there are no solutions, which proves the statement for bosons.

# Samuel L. Braunstein and A. Mann Measurement of the Bell operator and quantum teleportation Phys. Rev. A 51, R1727 - R1730 (1995)

Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel

# S. P. Walborn - W. A. T. Nogueira - S. Pádua - C. H. Monken, Optical Bell-state analysis in the coincidence basis,Europhys. Lett., 62 (2), p. 161 (2003)

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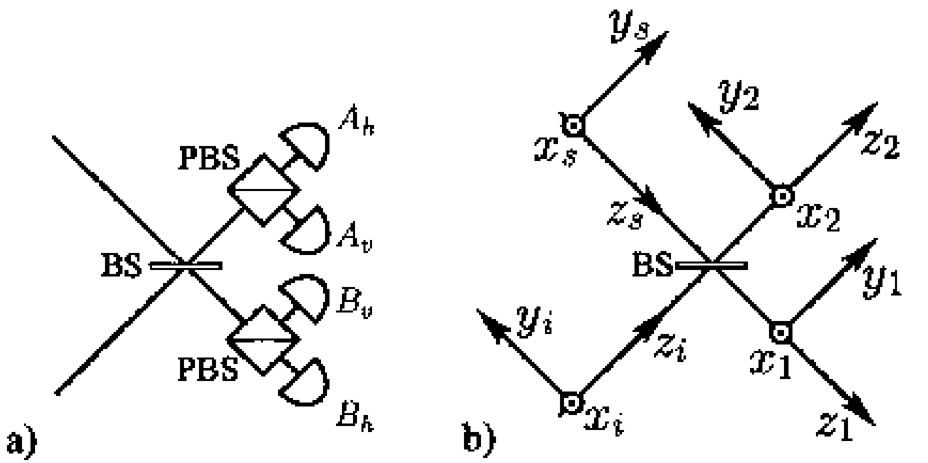
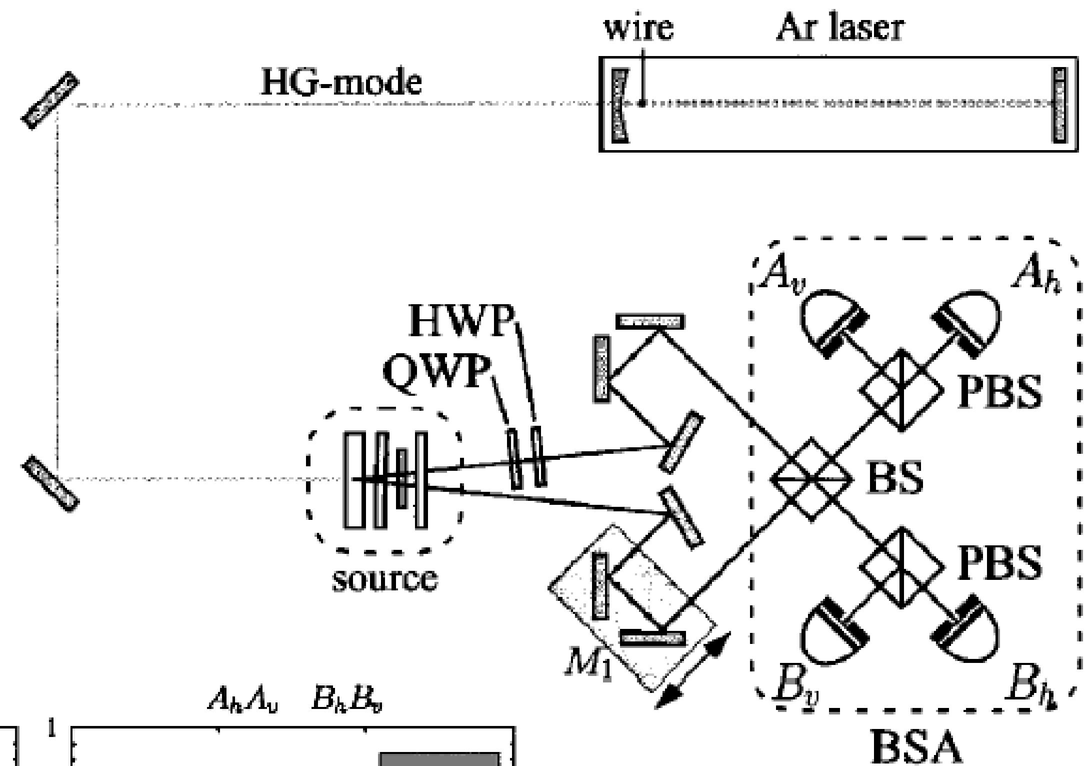
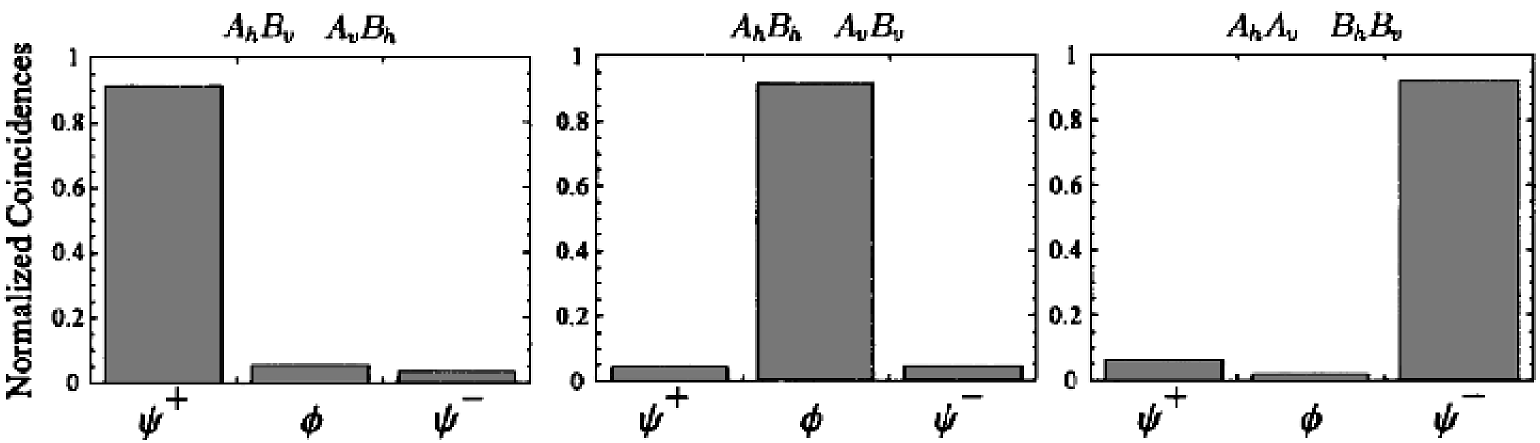
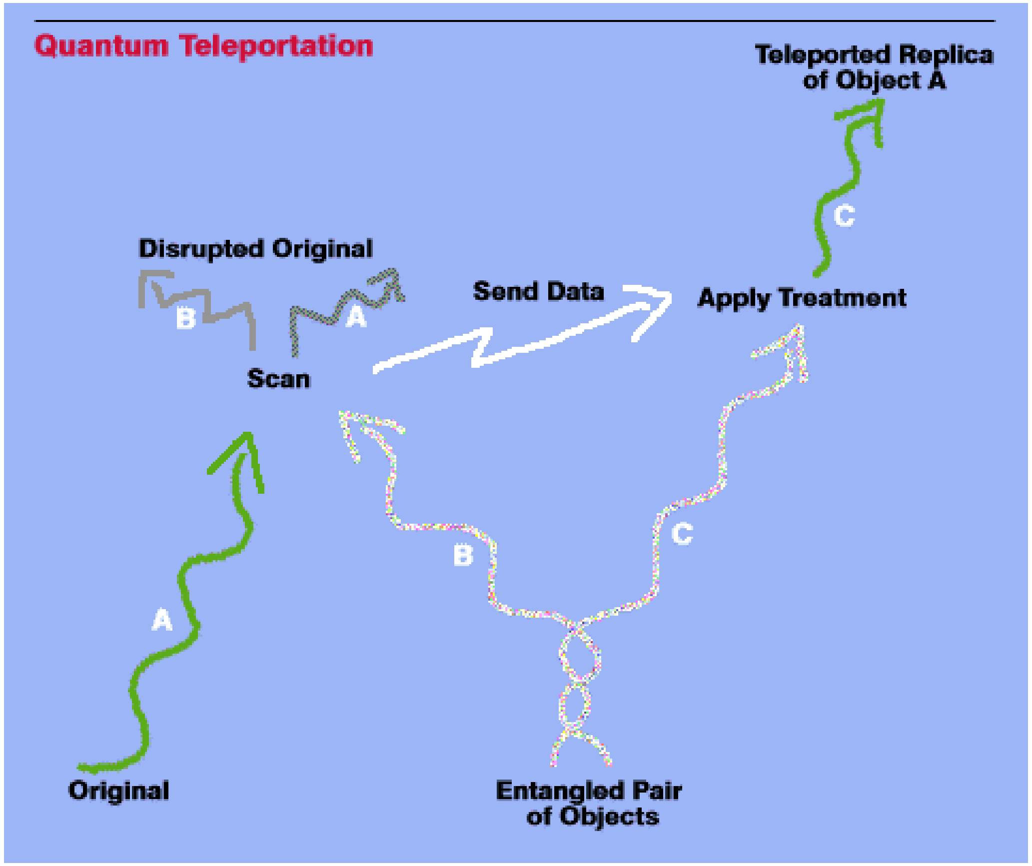
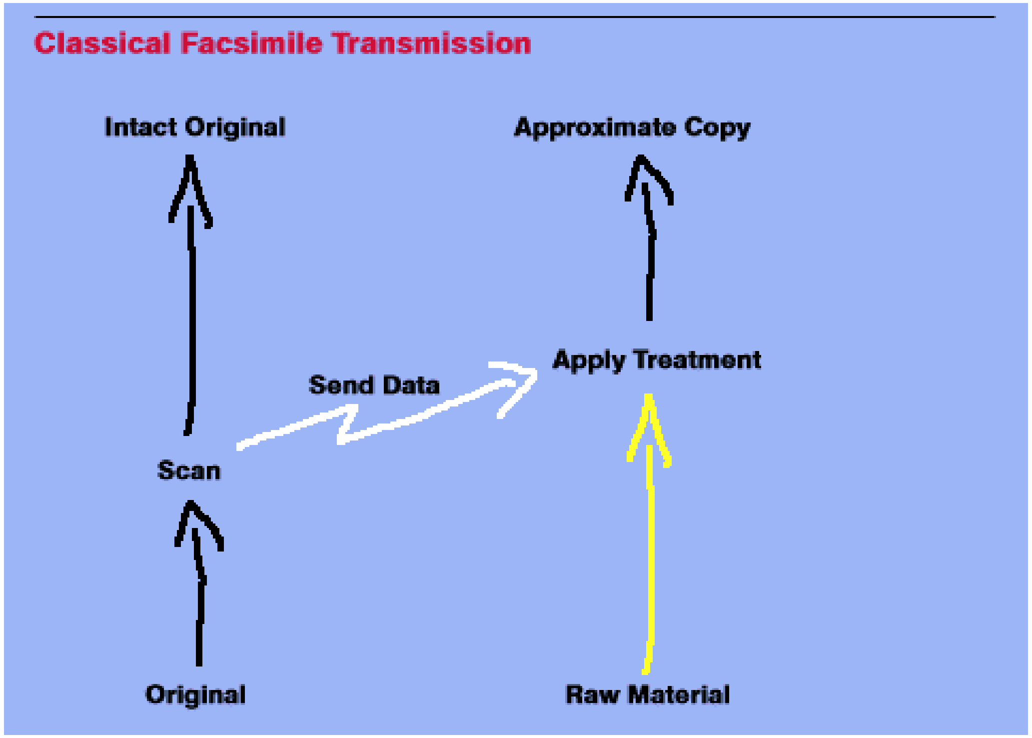


# Quantum Teleportation:
# First proposed in 1993 by Bennett et al.
# What do they mean by the science fiction term???



# Quantum Teleportation:

## First proposed in 1993 by Bennett et al.

**Alice wants to send Bob a qubit.**

**Three (+one) options (?):**

1. Physically send the qubit.
2. Broadcast this (quantum) information - Bob can obtain the information via some suitable receiver.
3. Measure the unknown qubit in his possession. The results of this measurement would be communicated to Bob, who then prepares a qubit in his possession accordingly, to obtain the desired state.

# Quantum Teleportation:

# First proposed in 1993 by Bennett et al.

**Alice wants to send Bob a qubit.**

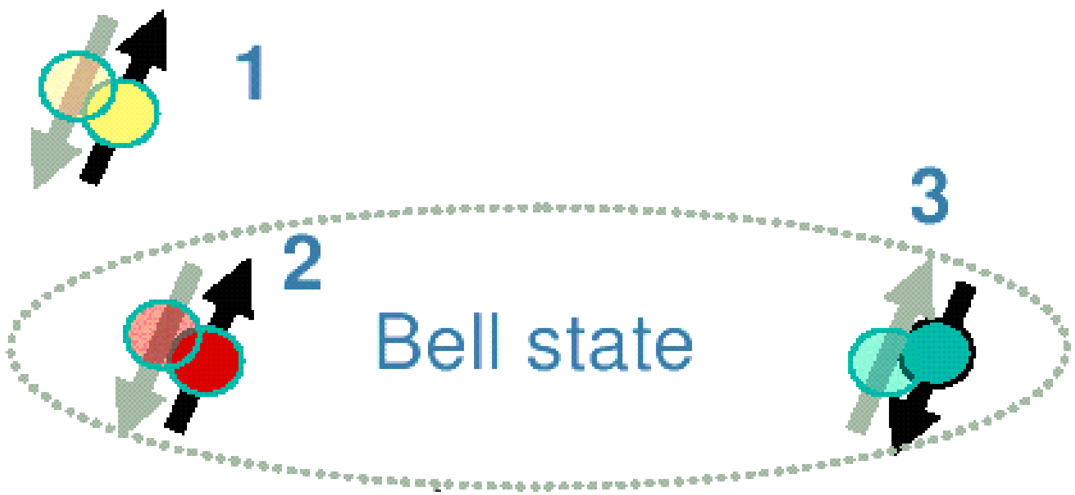
**Three problems :**

**1. Bad wires.**
**2. No-broadcast theorem - No-cloning theorem.**
**3. A quantum state is more then the sum of its' observables: without knowledge of the preparation procedure of a qubit, no measurement can determine its state.**

# Quantum Teleportation:

## A Scheme:

## Let Alice and Bob share a Bell state




**Alice takes the qubit to send (1) and the qubit from the Bell state (2) and measures them in the Bell basis**

**One of four possible outcomes ---> two bits of classical information**

**Send these bits to Bob, who operates on his qubit (3) a unitary transformation.**

# Quantum Teleportation:

# Now in Greek letters:

**Let us assume Alice and Bob are in the Bell state:**

**$|\Phi^{+}>_{2,3}$ then the whole system (1,2 and 3):**

$$|\Psi>_{1}\times|\Phi^{+}>_{2,3}=(\alpha|0>_{1}+\beta|1>_{1})\times 1/\sqrt{2}(|0>_{2}\times|0>_{3}+|1>_{2}\times|1>_{3})=$$

$$=...=1/2(|\Phi^{+}>_{1,2}\times(\alpha|0>_{3}+\beta|1>_{3})+|\Phi^{-}>_{1,2}\times(\alpha|0>_{3}-\beta|1>_{3})+$$

$$+|\Psi^{+}>_{1,2}\times(\beta|0>_{3}+\alpha|1>_{3})+|\Psi^{-}>_{1,2}\times(\beta|0>_{3}-\alpha|1>_{3}))$$

**so after Alices' measurement, Bob have equal probabilities for any of the 4 unitary transformations.**

# Quantum Teleportation:
# 4 possible outcomes:

$$|\Psi^+\rangle_{1,2} \text{ ---------} > \sigma_z|\Psi^+\rangle_3$$

$$|\Psi^-\rangle_{1,2} \text{ ---------} > \sigma_x|\Psi^+\rangle_3$$

$$|\Phi^+\rangle_{1,2} \text{ ---------} > I|\Psi^+\rangle_3$$

$$|\Phi^-\rangle_{1,2} \text{ ---------} > i\sigma_y|\Psi^+\rangle_3$$

# Quantum Teleportation:

## Remarks:

- After teleportation, Bob's qubit will take on the state $|\Psi>_3$, and Alice's qubit becomes part of an undefined entangled state. Teleportation does not result in the copying of qubits, and hence is consistent with the no cloning theorem.

- There is no transfer of matter or energy involved. Alice's particle has not been physically moved to Bob; only its state has been transferred.

No "Spocky" actions :)

Two necessary separated procedures:

1. Quantum Entanglement

2. Classical communication

# Quantum Teleportation:

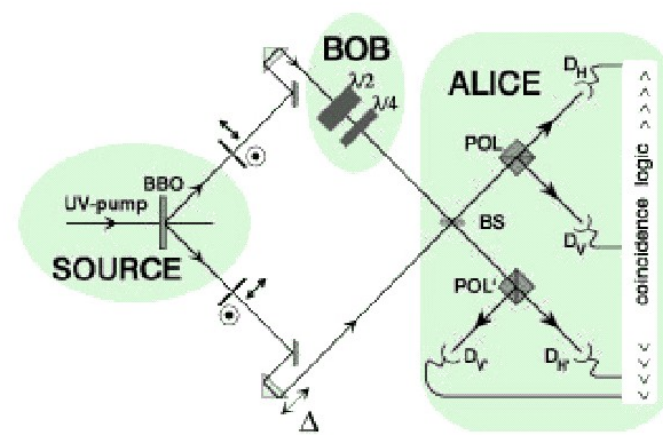
## Extensions:

- If we have *m* states system – we would need to use the right “rotations” (unitary transformation).

- Swapping : when particle 1 is part of an entangled pair: {0,1} – by teleporting the quantum information of particle 1 : particles 0 and 3 becomes entangled without even knowing each other.

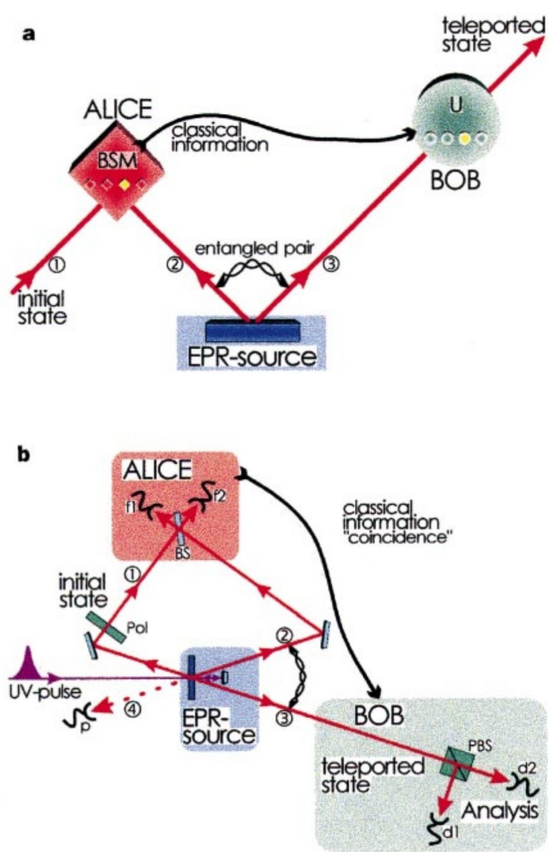
-Superdense coding

-Repeaters



# Quantum Teleportation: Experiments:

**D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Experimental Quantum Teleportation, Nature 390, 6660, 575-579 (1997).**



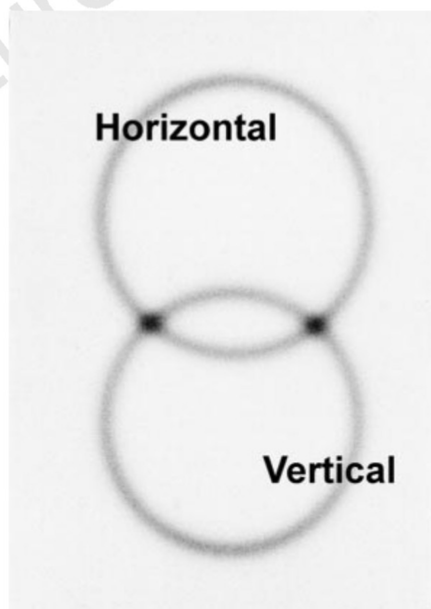




**Figure 2** Photons emerging from type II down-conversion (see text). Photograph taken perpendicular to the propagation direction. Photons are produced in pairs. A photon on the top circle is horizontally polarized while its exactly opposite partner in the bottom circle is vertically polarized. At the intersection points their polarizations are undefined; all that is known is that they have to be different, which results in entanglement.

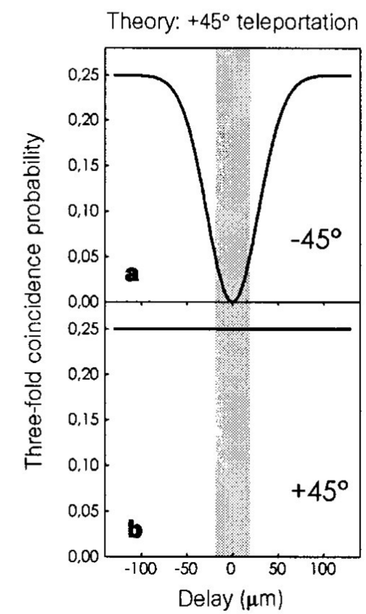



**Figure 3** Theoretical prediction for the three-fold coincidence probability betwe[en] the two Bell-state detectors (f1, f2) and one of the detectors analysing [the] teleported state. The signature of teleportation of a photon polarization state [at] +45° is a dip to zero at zero delay in the three-fold coincidence rate with [the] detector analysing −45° (d1f1f2) (**a**) and a constant value for the detector analy[sing] +45° (d2f1f2) (**b**). The shaded area indicates the region of teleportation.

# Quantum Teleportation: Experiments:

**D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Experimental Quantum Teleportation, Nature 390, 6660, 575-579 (1997).**

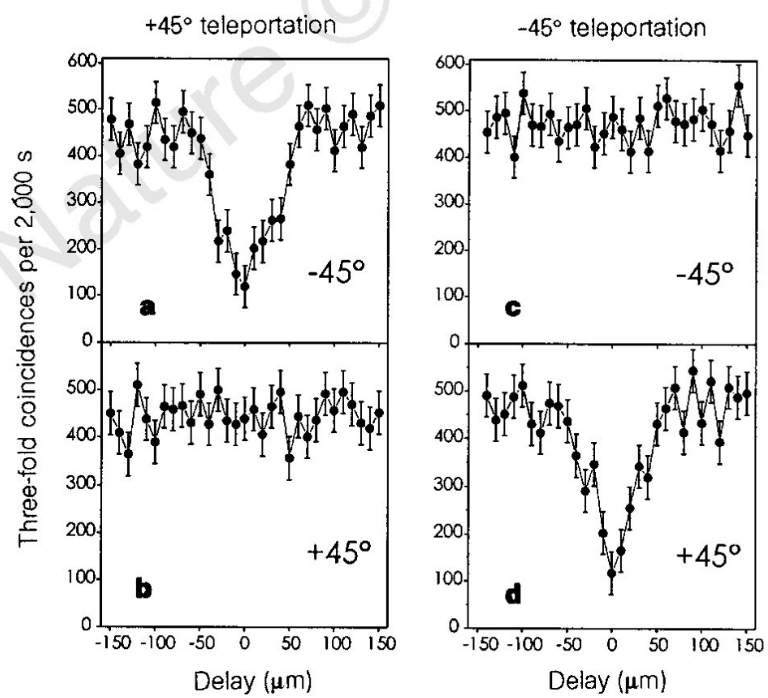



**Figure 4** Experimental results. Measured three-fold coincidence rates d1f1f2 (–45°) and d2f1f2 (+45°) in the case that the photon state to be teleported is polarized at +45° (**a** and **b**) or at –45° (**c** and **d**). The coincidence rates are plotted as function of the delay between the arrival of photon 1 and 2 at Alice's beam splitter (see Fig. 1b). The three-fold coincidence rates are plotted after subtracting the spurious three-fold contribution (see text). These data, compared with Fig. 3, together with similar ones for other polarizations (Table 1) confirm teleportation for an arbitrary state.

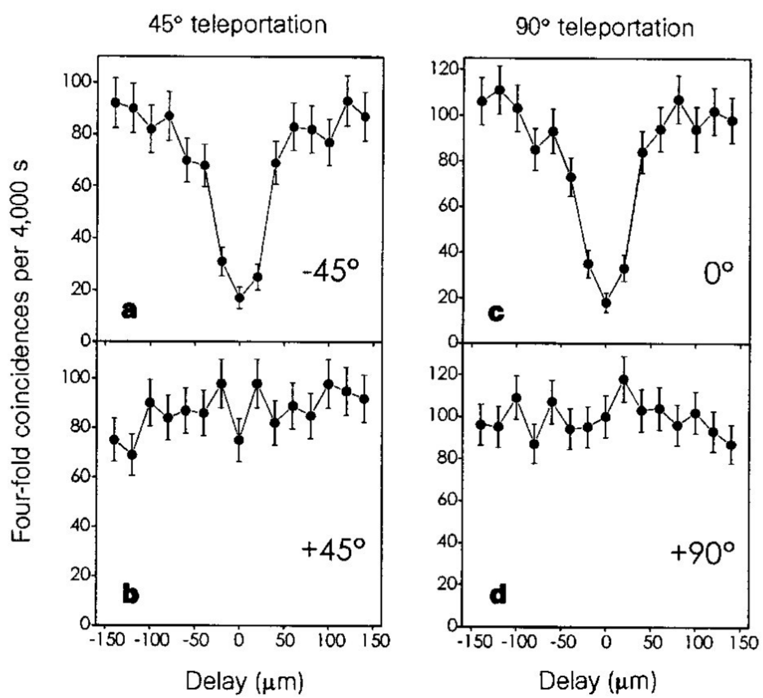



**Figure 5** Four-fold coincidence rates (without background subtraction). Conditioning the three-fold coincidences as shown in Fig. 4 on the registration of photon 4 (see Fig. 1b) eliminates the spurious three-fold background. **a** and **b** show the four-fold coincidence measurements for the case of teleportation of the +45° polarization state; **c** and **d** show the results for the +90° polarization state. The visibilities, and thus the polarizations of the teleported photons, obtained without any background subtraction are 70% ± 3%. These results for teleportation of two non-orthogonal states prove that we have demonstrated teleportation of the quantum state of a single photon.

# Quantum Teleportation:

# Experiments:

**J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Experimental Entangelment Swapping: Entangling Photons That Never Interact, Phys. Rev. Lett. 80, 18,3891-3894 (1998)**

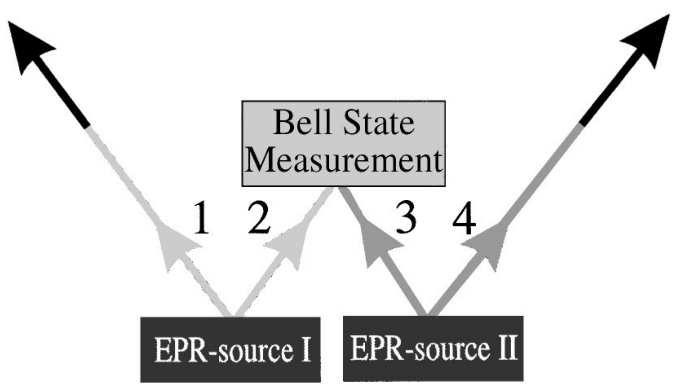



FIG. 1. Principle of entanglement swapping. Two EPR sources produce two pairs of entangled photons, pair 1-2 and pair 3-4. One photon from each pair (photons 2 and 3) is subjected to a Bell-state measurement. This results in projecting the other two outgoing photons 1 and 4 onto an entangled state. Change of the shading of the lines indicates the change in the set of possible predictions that can be made.

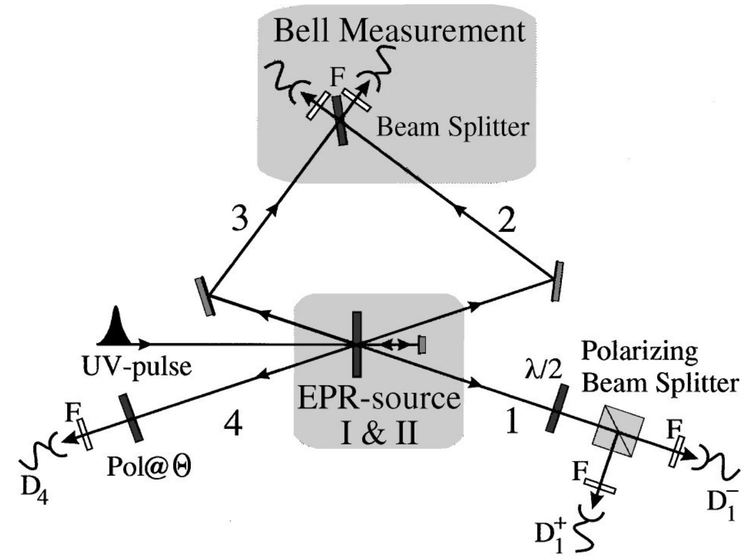



FIG. 2. Experimental setup. A UV pulse passing through a nonlinear crystal creates pair 1-2 of entangled photons. Photon 2 is directed to the beam splitter. After reflection, during its second passage through the crystal the UV pulse creates a second pair 3-4 of entangled photons. Photon 3 will also be directed to the beam splitter. When photons 2 and 3 yield a coincidence click at the two detectors behind the beam splitter, they are projected into the $|\Psi^-\rangle_{23}$ state. As a consequence of this Bell-state measurement the two remaining photons 1 and 4 will also be projected into an entangled state. To analyze their entanglement we look at coincidences between detectors $D_1^+$ and $D_4$, and between detectors $D_1^-$ and $D_4$, for different polarization angles $\Theta$. By rotating the $\lambda/2$ plate in front of the two-channel polarizer we can analyze photon 1 in any linear polarization basis. Note that, since the detection of coincidences between detectors $D_1^+$ and $D_4$, and $D_1^-$ and $D_4$ are conditioned on the detection of the $\Psi^-$ state, we are looking at fourfold coincidences. Narrow bandwidth filters (F) are positioned in front of each detector.

# Quantum Teleportation:

# Experiments:

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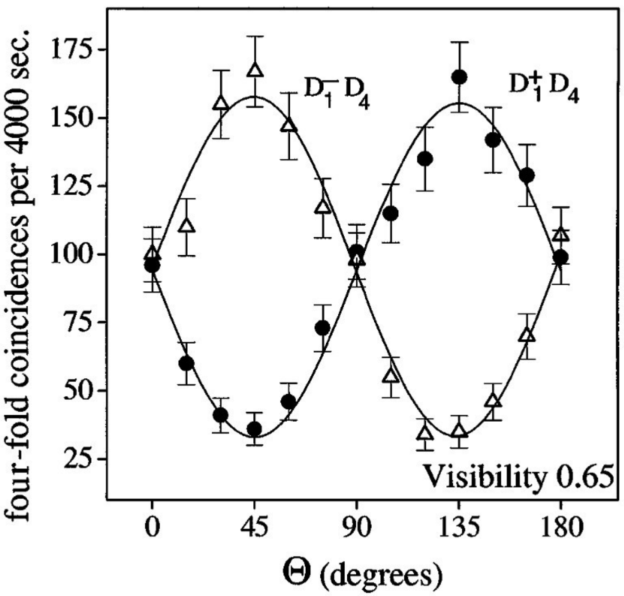


FIG. 3. Entanglement verification. Fourfold coincidences, resulting from twofold coincidence $D1^+D4$ and $D1^-D4$ conditioned on the twofold coincidences of the Bell-state measurement, when varying the polarizer angle Θ. The two complementary sine curves with a visibility of $0.65 \pm 0.02$ demonstrate that photons 1 and 4 are polarization entangled.

# Quantum Teleportation: Experiments:

**Y.-H. Kim, S. P. Kulik, and Y. Shih, Quantum Teleportation of a Polarization State with a Complete Bell State Measurement, Phys. Rev. Lett. 86, 1370 (2001)**

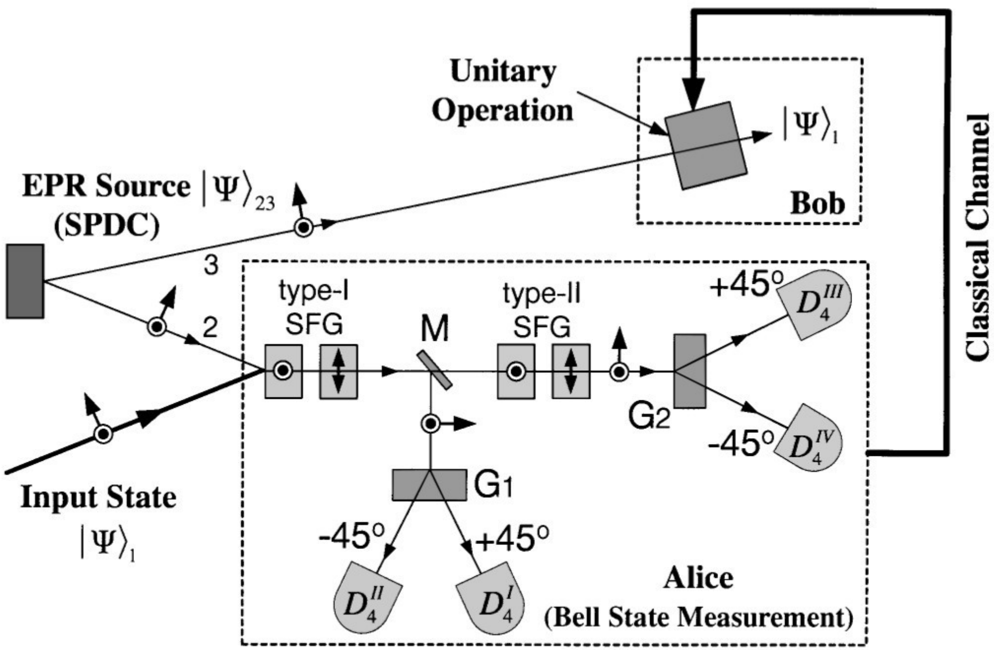


FIG. 1. Principle schematic of quantum teleportation with a complete BSM. Nonlinear interactions (SFG) are used to perform the BSM. ⊙ and ↕ represent the respective horizontal and vertical orientations of the optic axes of the crystals.

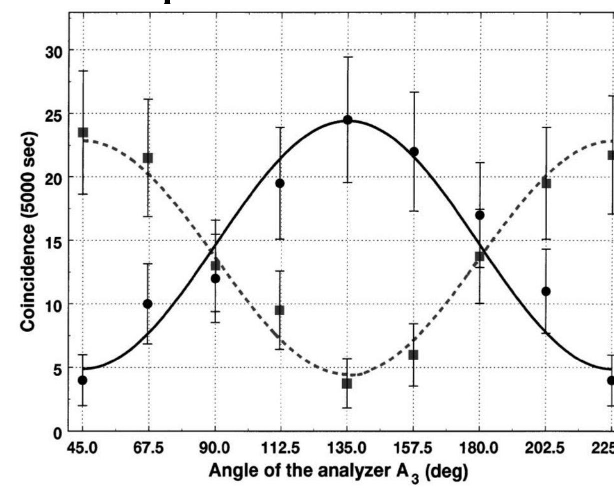


FIG. 2. The solid line (circled data points) is the joint detection rate $D_4^{\mathrm{I}}$-$D_3$ for 45° linear polarization as an input state. The dashed line (square data points) is for $D_4^{\mathrm{II}}$-$D_3$ for the same input state. The expected $\pi$ phase shift is clearly demonstrated.

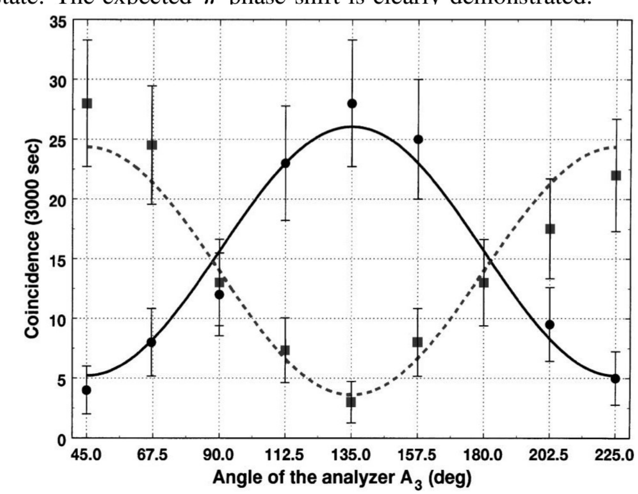


FIG. 3. The solid line (circled data points) is the joint detection rate $D_4^{\mathrm{III}}$-$D_3$ and the dashed line (square data points) is for $D_4^{\mathrm{IV}}$-$D_3$. Again, the expected $\pi$ phase shift is clearly demonstrated.

# Quantum Teleportation:

# Experiments:

**Y.-H. Kim, S. P. Kulik, and Y. Shih, Quantum Teleportation of a Polarization State with a Complete Bell State Measurement, Phys. Rev. Lett. 86, 1370 (2001)**

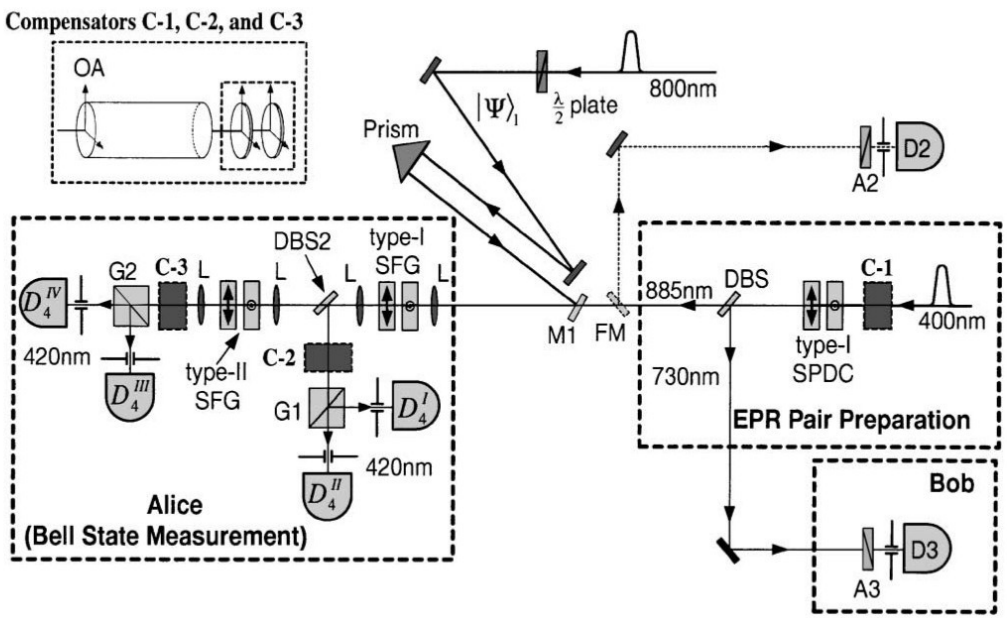


FIG. 4. Diagram of the experimental setup. The inset shows the details of the compensators. See text for details.

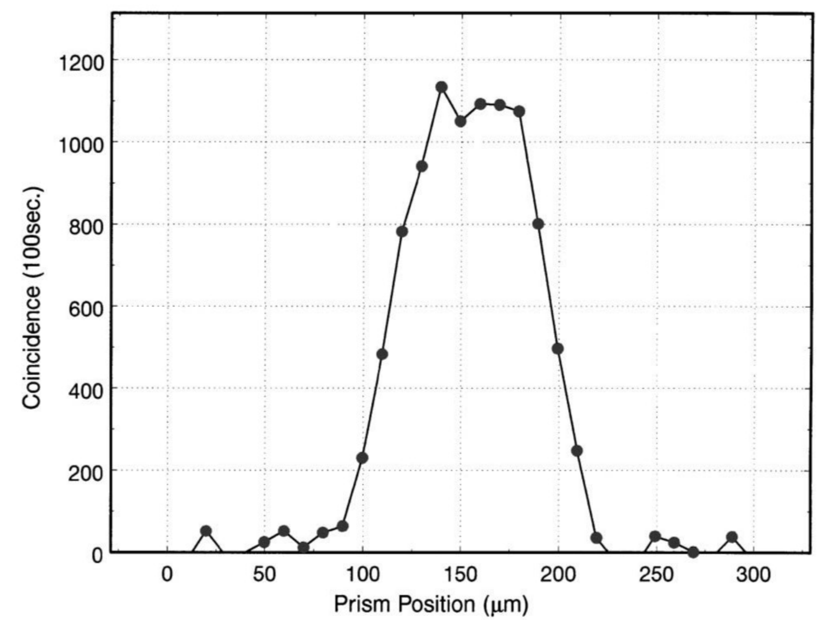


FIG. 5. SFG measurement as a function of the prism position. SFG is observed only when the input pulse and the SPDC photons overlap exactly inside the crystals.

# Theoretical proposal:

**C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W. K. Wootters, Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels, Phys. Rev. Lett. 70 1895-1899 (1993). This is the seminal paper that laid out the entanglement protocol.**

**L. Vaidman, Teleportation of Quantum States, Phys. Rev. A, (1994)**

**G. Brassard, S Braunstein, R Cleve, Teleportation as a Quantum Computation, Physica D 120 43-47 (1998)**

**G. Rigolin, Quantum Teleportation of an Arbitrary Two Qubit State and its Relation to Multipartite Entanglement, Phys. Rev. A 71 032303 (2005)**

# First experiments with photons:

**D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Experimental Quantum Teleportation, Nature 390, 6660, 575-579 (1997).**

**D. Boschi, S. Branca, F. De Martini, L. Hardy, & S. Popescu, Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual classical and Einstein-Podolsky-Rosen channels, Phys. Rev. Lett. 80, 6, 1121-1125 (1998)**

**J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Experimental Entangelment Swapping: Entangling Photons That Never Interact, Phys. Rev. Lett. 80, 18,3891-3894 (1998)**

**Y.-H. Kim, S. P. Kulik, and Y. Shih, Quantum Teleportation of a Polarization State with a Complete Bell State Measurement, Phys. Rev. Lett. 86, 1370 (2001)**

**I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, N. Gisin, Long-Distance Teleportation of Qubits at Telecommunication Wavelengths, Nature, 421, 509 (2003)**

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# First experiments with atoms:

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