Keynesian Economics Without the Phillips Curve

By Roger E.A. Farmer and Giovanni Nicoló*

Farmer (2012) develops a monetary search model (FM model) that describes the relationship between inflation, the output gap and the nominal interest rate in which the Phillips curve is replaced by a 'belief function'. We show that data simulated from the FM model is described by a Vector Error Correction Model, (VECM) as opposed to a Vector Autoregression (VAR) that characterizes the reduced form of the NK model. We develop an analog of the Taylor Principle for the FM model and we show that the conditions for local uniqueness of a rational expectations equilibrium fail to hold for empirically relevant parameters from U.S. data. We estimate the FM model on data from the United States and we show that it outperforms the New Keynesian model using a Bayesian model selection criterion.

U.S. macroeconomic data are well described by co-integrated non-stationary time series (Nelson and Plosser, 1982). This is true, not just of data that are growing such as GDP, consumption and investment; it is also true of data that are predicted by economic theory to be stationary such as the unemployment rate, the output gap, the inflation rate and the money interest rate, (King et al., 1991; Beyer and Farmer, 2007).[1]

Conventional New Keynesian (NK) theory cannot easily account for these facts. In the NK model; the inflation rate, the money interest rate and the output gap are described by a dynamic equilibrium path that converges to a unique equilibrium steady state. The reduced form representation of this model is a stationary Vector Auto-Regression (VAR), and, to account for a unit root, the NK model must assume that the natural rate of unemployment, or equivalently, the output gap, is itself a non-stationary process. Because there is a one-to-one mapping between the output gap and the difference of unemployment from its natural rate, we will move freely in our discussion between these two concepts.

Could the natural rate of unemployment be a random walk? Robert Gordon (2013) has argued that this is the case. We do not find that argument plausible. Because the natural rate of unemployment is associated with the solution to a social planning optimum, if persistent unemployment is caused by an increase in the natural rate of unemployment, high persistent unemployment is socially

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[1] A bounded random variable, such as the unemployment rate, cannot be a random walk over its entire domain. We view the I(1) assumption to be an approximation that is approximately valid for finite periods of time.

optimal. That is a possible explanation for the persistence of high unemployment following large recessions, but in our view it is unlikely. We do not think that the Great Depression was, in the words of Franco Modigliani, a "sudden bout of contagious laziness".

Farmer (2012) has proposed an alternative theory to the NK model that *can* explain persistent high unemployment. He calls this the Farmer Monetary (FM) model. The FM model differs from a canonical three-equation NK model by replacing the Phillips curve with the *belief function*. This is a new *fundamental* that has the same methodological status as preferences and technology.

Here, we study the dynamic properties of the FM model and we explain the role of the belief function in pinning down a unique equilibrium in an otherwise indeterminate model. In the NK model, equilibria are locally unique when the central bank follows a Taylor Rule in which the bank responds aggressively to inflation by raising the interest rate by more than 1% in response to a 1% increase in inflation. A central bank that responds in that way is said to follow the *Taylor Principal.*[2] We derive an analog of the Taylor Principle for the FM model and we compare parameter estimates of the FM model with parameter estimates of a canonical NK model. We show that our analog of the Taylor principle does not hold in U.S. data and we use that fact to explain the real effects of nominal shocks.

In the FM model, search frictions lead to the existence of multiple steady state equilibria and output and employment are demand determined. The belief function selects the period-by-period equilibrium, and, in the absence of shocks, initial conditions select the equilibrium to which the economy converges in the long-run. Because the model is otherwise under-determined, expectations can be both fundamental and rational in the sense of Muth (1961).

In the absence of the assumption that beliefs are fundamental, our theoretical model would exhibit both static and steady-state indeterminacy. Static indeterminacy means there are many possible equilibrium steady-state unemployment rates. Dynamic indeterminacy means there are many dynamic equilibrium paths, all of which converge to a given steady state.

We resolve static indeterminacy by assuming beliefs about future nominal income growth are fundamental. We resolve dynamic indeterminacy by assuming people react to nominal shocks by adjusting quantities, rather than prices. In our model, the covariance between nominal shocks and real economic activity is a parameter of the belief function.

The structural properties of the FM model translate into a critical property of its reduced form. Appealing to the Engle-Granger Representation theorem (Engle and Granger, 1987), we show that the FM model's reduced form is a co-integrated Vector Error Correction Model (VECM). The inflation rate, the output gap, and the federal funds rate, are non-stationary but display a common stochastic trend. Our model displays hysteresis; that is, in the absence of stochastic shocks, the

[2]Woodford (2003b, page 90).

steady-state of the model depends on initial conditions.

Previous studies have focused on the change in the high frequency properties of data. Richard Clarida, Jordi Galí and Mark Gertler (2000) argued that prior to 1979Q3, the Fed had operated a passive interest rate rule in which the Federal Open Market Committee (FOMC) raised the fed funds rate by less than 1% in response to a 1% increase in the expected inflation rate. After 1983Q1 they switched to a rule where policy was more aggressive; they raised the funds rate rate by more than 1% in response to a 1% increase in expected inflation.

The Clarida-Galí-Gertler (CGG) paper is conducted in partial equilibrium. CGG estimate the policy rule but calibrate the other parameters of their model. Work by Thomas Lubik and Frank Schorfheide (2004) has confirmed the CGG results in a fully specified Dynamic Stochastic General Equilibrium (DSGE) model. Their study is, however, unable to address the low frequency properties of the data, because they remove these low-frequency components using the Hodrick-Prescott filter. That leads to the open question; if one were to estimate the New-Keynesian (NK) model using data that has not been detrended in this way, how would the NK model stack up against the FM model? We address that question in this paper.

We estimate the parameters of the FM model and of a canonical NK model using post-War U.S. data on the inflation rate, the output gap and the federal funds rate, and we compare the values of the posterior likelihoods of the two models using Bayesian methods. We find that the posterior odds ratio favors the FM model. We explain our findings by appealing to the theoretical properties of the two models. The data favor a reduced-form model that is described by a VECM as opposed to a VAR.

I. The Structural Forms of the NK and FM Models

In Section I we write down the two structural models that form the basis for our empirical estimates in Section V These models have two equations in common. One of these is a generalization of the NK IS curve that arises from the Euler equation of a representative agent. The other is a policy rule that describes how the Fed sets the fed funds rate. The two common equations of our study are described below.

A. Two Equations that the NK and FM Models Share in Common

We assume the log of potential real GDP grows at a constant rate and the difference of the log of observed real GDP from the log of potential real GDP is an I(1) series.[3] We estimate this series in a first stage, by regressing the log of real GDP on a constant and a time trend. The residual series is our empirical analog of the output gap. Our theoretical model implies that the output gap should be

[3]A series is I(k) if the k'th difference of the series is covariance stationary (Hamilton, 1994).

non-stationary and cointegrated with the CPI inflation rate and the federal funds rate.

In Equations (1) and (2), y_t is our constructed output-gap measure, R_t is the federal funds rate and π_t is the CPI inflation rate. The term $z_{d,t}$ is a demand shock, $z_{R,t}$ is a policy shock and $z_{s,t}$ is a random variable that represents the Fed's estimate of potential GDP.[4]

$$(1)\quad ay_t - a\mathbb{E}_t(y_{t+1}) + [R_t - \mathbb{E}_t(\pi_{t+1})] \\ = \eta\left(ay_{t-1} - ay_t + [R_{t-1} - \pi_t]\right) + (1-\eta)\rho + z_{d,t}.$$

$$(2)\qquad R_t = (1-\rho_R)\bar{r} + \rho_R R_{t-1} + (1-\rho_R)\left[\lambda\pi_t + \mu\left(y_t - z_{s,t}\right)\right] + z_{R,t}.$$

Equation (1) is a generalization of the dynamic IS curve that appears in standard representations of the NK model. In the special case when $\eta = 1$ this equation can be derived from the Euler equation of a representative agent.[5] An equation of this form for the general case when $\eta \neq 1$ can be derived from a heterogeneous agent model (Farmer, 2016) where the lagged real interest rate captures the dynamics of borrowing and lending between patient and impatient groups of people. In the case when $\eta = 1$, the parameter a is the inverse of the intertemporal elasticity of substitution and ρ is the time preference rate.

Equation (2) is a *Taylor Rule* (Taylor, 1999) that represents the response of the monetary authority to the lagged nominal interest rate, the inflation rate and the output gap. The monetary policy shock, $z_{R,t}$, denotes innovations to the nominal interest rate caused by unpredictable actions of the monetary authority. The parameters ρ_R, λ and μ are policy elasticities of the fed funds rate with respect to the lagged fed funds rate, the inflation rate and the output gap.

B. Two Equations that Differentiate the Two Models

The third equation of the NK model is given by

$$(3.a)\qquad \pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \phi\left(y_t - z_{s,t}\right).$$

Here, β is the discount rate of the representative person and ϕ is a compound parameter that depends on the frequency of price adjustment. Since β is expected to be close to one, we will impose the restriction $\beta = 1$ when discussing the theoretical properties of the model. This restriction implies that the long-run Phillips curve is vertical. If instead, $\beta < 1$, the NK model has an upward sloping long-run Phillips curve in inflation output-gap space. An extensive literature

[4]More precisely, $z_{s,t}$ is the Fed's estimate of the deviation of the log of potential GDP from a linear trend.

[5]See for example Galí (2008), or Woodford (2003a).

derives the NK Phillips curve from first principles, see for example Galí (2008), based on the assumption that frictions of one kind or another prevent firms from quickly changing prices in response to changes in demand or supply shocks.

In contrast to the NK Phillips curve, the FM model is closed by a belief function (Farmer, 1999). The functional form for the belief function that we use in this study is described by Equation (3.b),

$$\mathbb{E}_t \left[x_{t+1}\right] = x_t, \tag{3.b}$$

where $x_t \equiv \pi_t + (y_t - y_{t-1})$ is the growth rate of nominal GDP.

The belief function is a mapping from current and past observable variables to probability distributions over future economic variables. In the functional form we use here, it asserts that agents forecast that future nominal GDP growth will equal current nominal GDP growth; that is, nominal GDP growth is a martingale. Farmer (2012) has shown that this specification of beliefs is a special case of adaptive expectations in which the weight on current observations of GDP growth is equal to 1.[6]

In the FM model, the monetary authority chooses whether changes in the current growth rate of nominal GDP will cause changes in the expected inflation rate or in the output gap. Importantly, these changes will be permanent. The belief function, interacting with the policy rule, selects how demand and supply shocks are distributed between permanent changes to the output gap, and permanent changes to the expected inflation rate.

II. The Steady-State Properties of the Two Models

In this section we compare the theoretical properties of the non-stochastic steady-state equilibria of the NK and FM models. The NK model has a unique steady state. The FM model, in contrast, has a continuum of non-stochastic steady state equilibria. Which of these equilibria the economy converges to depends on the initial condition of a system of dynamic equations. In the physical sciences, this property is known as *hysteresis*.[7]

Rather than treat the multiplicity of steady state equilibria as a deficiency, as is often the case in economics, we follow Farmer (1999) by defining a new fundamental, the *belief function*. When the model is closed in this way, equilibrium uniqueness is restored and every sequence of shocks is associated with a unique sequence of values for the three endogenous variables.

We begin by shutting down shocks and describing the theoretical properties of the steady-state of the NK model. The values of the steady-state inflation rate, interest rate and output gap in the NK model are given by the following equations

[6]Farmer (2012) allowed for a more general specification of adaptive expectations and he found that the data favor the special case we use here.

[7]This analysis reproduces the discussion from Farmer (2012) and we include it here for completeness. Models that display hysteresis were introduced to economics by Blanchard and Summers (1986, 1987).

$$\bar{\pi} = \frac{\phi(\bar{r} - \rho)}{\phi(1-\lambda) - \mu(1-\beta)}, \qquad \bar{R} = \rho + \bar{\pi}, \qquad \bar{y} = \bar{\pi}\frac{(1-\beta)}{\phi}. \tag{3}$$

When $\beta < 1$, the long-run Phillips curve, in output-gap inflation space, is upward sloping. As β approaches 1, the slope of the long-run Phillips curve becomes vertical and these equations simplify as follows,

$$\bar{\pi} = \frac{(\bar{r} - \rho)}{(1-\lambda)}, \qquad \bar{R} = \rho + \bar{\pi}, \qquad \bar{y} = 0. \tag{4}$$

For this canonical special case, the steady state of the NK model is defined by Equations (4).

Contrast this with the steady state of FM model, which has only two steady state equations to solve for three steady state variables. These are given by the steady state version of the IS curve, Equation (1), and the steady state version of the Taylor Rule, Equation (2).

The FM model is closed, not by a Phillips Curve, but by the belief function. In the specific implementation of the belief function in this paper we assume that beliefs about future nominal income growth follow a martingale. This equation does not provide any additional information about the non-stochastic steady state of the model because the same variable, steady-state nominal income growth, appears on both sides of the equation.

Solving the steady-state versions of equations (1) and (2) for $\bar{\pi}$ and $\bar{R}$ as a function of $\bar{y}$ delivers two equations to determine the three variables, $\bar{\pi}$, $\bar{R}$ and $\bar{y}$.

$$\bar{\pi} = \frac{(\bar{r} - \rho)}{(1-\lambda)} + \frac{\mu}{(1-\lambda)}\bar{y}, \qquad \bar{R} = \rho + \bar{\pi}. \tag{5}$$

The fact that there are only two equations to determine three variables implies that the steady-state of the FM model is under determined. We refer to this property as *static indeterminacy.* Static indeterminacy is a source of endogenous persistence that enables the FM model to match the high persistence of the unemployment rate in data and it implies that the reduced form representation of the FM model is a VECM, as opposed to a VAR.

An implication of the static indeterminacy of the model is that policies that affect aggregate demand have permanent long-run effects on the output gap and the unemployment rate. In contrast, the NK model incorporates the NRH, a feature which implies that demand management policy cannot affect real economic activity in the long-run.

III. The Dynamic Properties of the Two Models

In this section we discuss the NK Taylor Principal and we derive an analog of this principal for the FM model. For both the NK and FM models we study the special case of $\rho_R = 0$, and $\eta = 0$. The first of these restrictions sets the response of the Fed to the lagged interest rate to zero. The second restricts the IS curve to the representative agent case. These restrictions allow us to generate, and compare, analytical expressions for the Taylor principal in both models.

The special cases of Equations (1) and (2) are given by

$$ay_t = aE_t(y_{t+1}) - (R_t - \mathbb{E}_t(\pi_{t+1})) + \rho + z_{d,t}, \tag{1'}$$

and

$$R_t = \bar{r} + \lambda\pi_t + \mu\,(y_t - z_{s,t}) + z_{R,t}. \tag{2'}$$

The Taylor Principle directs the central bank to increase the federal funds rate by more than one-for-one in response to an increase in the inflation rate. When the Taylor Principle is satisfied, the dynamic equilibrium of the NK model is locally unique. When that property holds, we say that the unique steady state is locally determinate (Clarida et al., 1999).

When the central bank responds only to the inflation rate, the Taylor principle is sufficient to guarantee local determinacy. When the central bank responds to the output gap as well as to the inflation rate, a sufficient condition for the NK model to be locally determinate is that

$$\left|\lambda + \frac{1-\beta}{\phi}\mu\right| > 1. \tag{6}$$

In Appendix A we derive this result analytically and we compare it with the dynamic properties of the FM model. There, we establish that the FM model is characterized by an analog of the Taylor Principle. For the special case of logarithmic preferences, that is, when $a = 1$, a sufficient condition for local determinacy is,

$$\left|\frac{\lambda}{\lambda - \mu}\right| > 1. \tag{7}$$

This condition guarantees that the *set* of steady state equilibria model is dynamically determinate and it is the FM analog of the Taylor Principal. It requires the interest-rate response of the central bank to changes in inflation to be sufficiently large relative to its response to changes in the output gap.

When the representative agent has CRA preferences with $a \neq 1$, the condition is more complicated and we are unable to find an analytic expression for the FM analog of the Taylor Principal. We are, however, able to find an analytic condition for the case when $\lambda = \mu$. In this special case, the Taylor principal fails whenever

$$a < 1 + \frac{\lambda}{2}. \tag{8}$$

The model always has a root of zero and a root of unity. When $\lambda = \mu = 0.7$, the determinacy condition fails when a is larger than 1.35. When λ and μ are different and are chosen to equal our estimated values the model displays dynamic indeterminacy for positive values of a that are greater than, but much closer to, one. This case, drawn for values of $\eta = 0.89$, $\rho = 0.021$, and $\rho_R = 0.98$ is depicted in Figure 1.

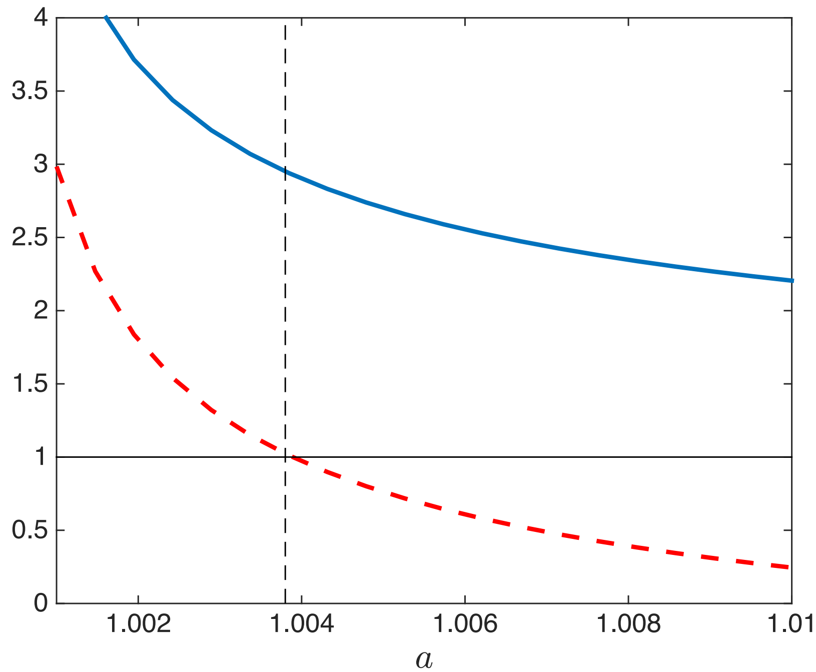

Figure 1. : Characteristic roots as a function of a: $\lambda = 0.76$, $\mu = 0.75$

We conclude from our analysis of the roots that plausibly parametrized versions of the FM model display dynamic as well as static indeterminacy. That conclusion is confirmed by our empirical estimates, described in Section V.

The conjunction of static and dynamic indeterminacy provide two sources of endogenous persistence. Static indeterminacy implies that the output-gap contains an I(1) component. Instead of converging to a point in interest-rate/inflation/output-gap space, the data converge to a one-dimensional linear manifold. Dynamic indeterminacy implies that the fed funds rate, the inflation rate and the unemployment rate display persistent deviations from this manifold.

The fact that the model displays dynamic indeterminacy allows us to explain the fact that prices appear to move slowly in data. In response to a purely monetary shock, there is an equilibrium path in which prices are predetermined and the output gap falls in response to an increase in the fed funds rate. In this equilibrium, prices are not sticky in the sense that there is a cost or barrier to price adjustment. Instead, as in Farmer (1991), prices are sticky because of the way people forecast the future and the covariances of prices with contemporaneous shocks determine the degree of price stickiness. We treat these covariances as fundamental parameters of the belief function and we set them to zero in our estimation of the FM model.[8]

IV. Solving the NK and FM Models

A. Finding the Reduced forms of the Two Models

Sims (2001) showed how to write a structural DSGE model in the form

$$\Gamma_0 X_t = C + \Gamma_1 X_{t-1} + \Psi \varepsilon_t + \Pi \eta_t \tag{9}$$

where $X_t \in \mathbb{R}^n$ is a vector of variables that may or may not be observable. Using the following definitions, the NK and FM models can both be expressed in this way,

$$X_t = \begin{bmatrix} y_t \\ \pi_t \\ R_t \\ \mathbb{E}_t(y_{t+1}) \\ \mathbb{E}_t(\pi_{t+1}) \\ z_{d,t} \\ z_{s,t} \end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix} z_{R,t} \\ \varepsilon_{d,t} \\ \varepsilon s, t \end{bmatrix}, \qquad \eta_t = \begin{bmatrix} y_t - \mathbb{E}_{t-1}(y_t) \\ \pi_t - \mathbb{E}_{t-1}(\pi_t) \end{bmatrix}. \tag{10}$$

The shocks ε_t are called *fundamental* and the shocks η_t are *non-fundamental*. By exploiting a property of the generalized Schur decomposition (Gantmacher, 2000) Sims provided an algorithm, GENSYS, that determines if there exists a VAR of the form

$$X_t = \hat{C} + G_0 X_{t-1} + G_1 \varepsilon_t, \tag{11}$$

such that all stochastic sequences $\{X_t\}_{t=1}^{\infty}$ generated by this equation also satisfy the structural model, Equation (9).[9] To guarantee that solutions remain bounded, all of the eigenvalues of G_0 must lie inside the unit circle. When a solution of this

[8]Since the model has four shocks, but only three observable variables, setting two co-variance terms to zero is an identification restriction.

[9]The generalized Schur decomposition exploits the properties of the generalized eigenvalues of the matrices $\{\Gamma_0, \Gamma_1\}$.

kind exists we refer to it as a reduced form of (9).

If a reduced form exists, it may or may not be unique. GENSYS reports on whether a reduced form exists and, if it exists, whether it is unique. The algorithm eliminates unstable generalized eigenvalues of the matrices $\{\Gamma_0, \Gamma_1\}$ by finding expressions for the non-fundamental shocks, η_t, as functions of the fundamental shocks, ε_t. When there not enough unstable generalized eigenvalues, there are many candidate reduced forms.

For the case of multiple candidate reduced forms, Farmer et al. (2015) show how to redefine a subset of the non-fundamental shocks as new fundamental shocks. For example, if the model has one degree of indeterminacy, one may define a vector of *expanded fundamental shocks*, $\hat{\varepsilon}_t$,

$$\hat{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_t \\ \eta_t^1 \end{bmatrix} \tag{12}$$

The parameters of the variance-covariance matrix of expanded fundamental shocks are fundamentals of the model that may be calibrated or estimated in the same way as the parameters of the utility function or the production function.

In the FM model, we assume that prices are set one period in advance and under this definition of expanded fundamentals our model has a unique reduced form. To solve and estimate both the NK and FM models, we use an implementation of GENSYS, (Sims, 2001) programmed in DYNARE (Adjemian et al., 2011), to find the reduced form associated with any given point in the parameter space and we use the Kalman filter to generate the likelihood function and a Markov Chain Monte Carlo algorithm to explore the posterior.

B. *An Important Implication of the Engle-Granger Representation Theorem*

The reduced form of both the NK and FM models is a Vector-Auto-Regression with form of Equation (11). We reproduce that equation below.

$$X_t = \hat{C} + G_0 X_{t-1} + G_1 \varepsilon_t. \tag{11$'$}$$

Robert Engle and Clive Granger (1987) showed how to rewrite a Vector-Autoregression in the equivalent form

$$\Delta X_t = \hat{C} + \hat{\Pi} X_{t-1} + G_1 \varepsilon_t, \tag{13}$$

where $X_t \in \mathbb{R}^n$. If the matrix $\hat{\Pi}$ has rank n, this system of equations has a well defined non-stochastic steady state, $\bar{X}$, defined by shutting down the shocks and setting $X_t = \bar{X}$ for all t. $\bar{X}$ is defined by the expression,

$$\bar{X} = -\hat{\Pi}^{-1} \hat{C}. \tag{14}$$

When $\hat{\Pi}$ has rank $m < n$, it can be written as the product of an $n \times k$ matrix

α and a $k \times n$ matrix $\beta^\top$, where $k \equiv n - m$,

$$\hat{\Pi} = \alpha\beta^\top. \tag{15}$$

When $\hat{\Pi}$ has reduced rank, there is no steady state. This begs the question; what is the behavior of sequences $\{X_t\}_{t=1}^{\infty}$ generated by Equation (13)?

If we set $\varepsilon_t = 0$ for all t, the sequence X_t will converge to a point on an $n - m$ dimensional linear subspace of $\mathbb{R}^n$ that depends on the starting point X_0. The rows of α are referred to as loading factors, and the columns of β are called co-integrating vectors.[10] This discussion demonstrates the connection between the existence of a unique solution to the steady state equations of a model and the representation of the reduced form.

The NK model has a unique steady state defined by the solution to equations (4). In contrast, the FM model has only two steady state equations, (5), to define the three steady state variables, $\bar{y}$, $\bar{\pi}$, and $\bar{R}$. When we use the Engle-Granger representation theorem to write the NK model in the form of equation (13), the matrix $\hat{\Pi}$ has full rank. The equivalent matrix for the FM representation has reduced rank and consequently the reduced form of the FM model is a VecM as opposed to a VAR.

V. Estimating the Parameters of the NK and FM Models

In this section we estimate the NK and FM models. Both models share equations (1) and (2) in common. We reproduce these equations below for completeness.

$$\begin{aligned} ay_t - a\mathbb{E}_t(y_{t+1}) + [R_t - \mathbb{E}_t(\pi_{t+1})] \\ = \eta\left(ay_{t-1} - ay_t + [R_{t-1} - \pi_t]\right) + (1-\eta)\rho + z_{d,t}. \end{aligned} \tag{16}$$

$$R_t = (1-\rho_R)\bar{r} + \rho_R R_{t-1} + (1-\rho_R)\left[\lambda\pi_t + \mu\left(y_t - z_{s,t}\right)\right] + z_{R,t}. \tag{17}$$

For the NK model these equations are supplemented by the Phillips curve, Equation (3.a),

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \phi\left(y_t - z_{s,t}\right), \tag{3.a}$$

and for the FM model they are supplemented by the belief function, Equation (3.b),

$$\mathbb{E}_t\left[x_{t+1}\right] = x_t. \tag{3.b}$$

[10] The co-integrating vectors are not uniquely defined; they are linear combinations of the steady state equations of the non-stochastic model.

We assume in both models that the demand and supply shocks follow autoregressive processes that we model with equations (18) and (19),

$$z_{d,t} = \rho_d z_{d,t-1} + \varepsilon_{d,t}, \tag{18}$$

$$z_{s,t} = \rho_s z_{s,t-1} + \varepsilon_{s,t}. \tag{19}$$

Figure 2 plots the data that we use to compare the models. We use three time series for the U.S. over the period from 1954Q3 to 2007Q4: the effective Federal Funds Rate, the CPI inflation rate and the percentage deviation of real GDP from a linear trend.

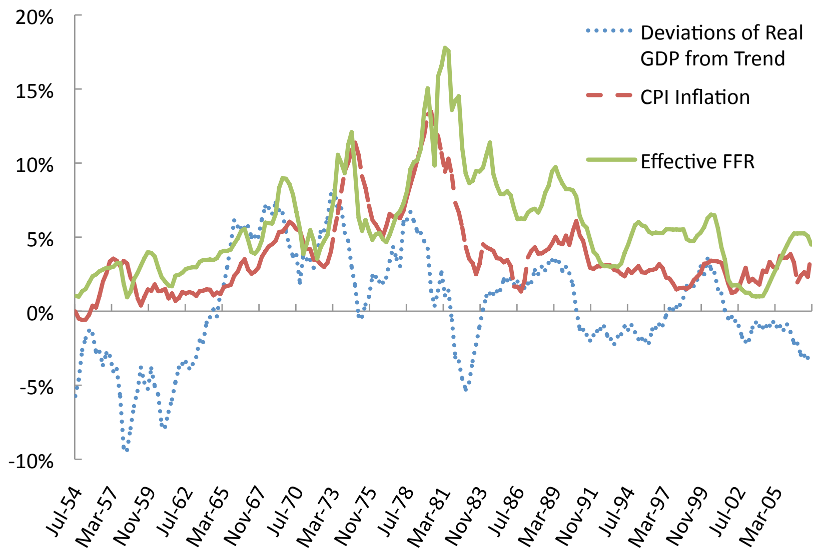

Figure 2. : U.S. data

Source: FRED, Federal Reserve Bank of St. Louis.

To estimate the models, we used a Markov-Chain Monte-Carlo algorithm, implemented in DYNARE (Adjemian et al., 2011). Formal tests reject the null of parameter constancy over the entire period. Beyer and Farmer (2007) find evidence of a break in 1980 and we know from the Federal Reserve Bank's own website (of San Francisco, January 2003) that the Fed pursued a monetary targeting strategy from 1979Q3 through 1982Q3. For this reason, and in line with previous studies, (Clarida et al., 2000; Lubik and Schorfheide, 2004; Primiceri, 2005) we estimated both models over two separate sub-periods.

Our first sub-period runs from 1954Q3 through 1979Q2. The beginning date is one year after the end of the Korean war; the ending date coincides with the appointment of Paul Volcker as Chairman of the Federal Reserve Board. We excluded the period from 1979Q3 through 1982Q4 because, over that period, the Fed was explicitly targeting the growth rate of the money supply. In 1983Q1, it reverted to an interest rate rule.

Our second sub-period runs from 1983Q1 to 2007Q4. We ended the sample with the Great Recession to avoid potential issues arising from the fact that the federal funds rate hit a lower bound in the beginning of 2009 and our linear approximation is unlikely to fare well for that period.

Table 1a summarizes the prior parameter distributions that we used in this procedure for those parameters that were the same in both sub-samples. The table reports the prior shape, mean, standard deviation and 90% probability interval. Table 1b presents the prior distributions for parameters that were different in the two subsamples. These were λ, the policy coefficient for the interest rate response to the inflation rate, and σ_s, the standard deviation of the supply shock.

We set $\lambda = 0.9$ in the first sub-period and $\lambda = 1.1$ in the second. We chose these values because Lubik and Schorfheide (2004) found that policy was indeterminate in the first period and determinate in the second. These choices ensure that our priors are consistent with these differences in regimes.

Table 1.A: Prior distribution, common model parameters

Name	Range	Density	Mean	Std. Dev.	90% interval
a	R^+	*Gamma*	3.5	0.50	[2.67,4.32]
ρ	R^+	*Gamma*	0.02	0.005	[0.012,0.028]
η	$[0,1)$	*Beta*	0.85	0.10	[0.65,0.97]
$\bar{r}$	R^+	*Uniform*	0.05	0.029	[0.005,0.095]
ρ_R	$[0,1)$	*Beta*	0.85	0.10	[0.65,0.97]
μ	R^+	*Gamma*	0.70	0.20	[0.41,1.06]
ρ_d	$[0,1)$	*Beta*	0.80	0.05	[0.71,0.87]
ρ_s	$[0,1)$	*Beta*	0.90	0.05	[0.81,0.97]
σ_R	R^+	*Inverse Gamma*	0.01	0.003	[0.005,0.015]
σ_d	R^+	*Inverse Gamma*	0.01	0.003	[0.005,0.015]
σ_ζ	R^+	*Inverse Gamma*	0.005	0.003	[0.002,0.010]
ρ_{ds}	[-1,1]	*Uniform*	0	0.58	[-0.9,0.9]
ρ_{dR}	[-1,1]	*Uniform*	0	0.58	[-0.9,0.9]
ρ_{sR}	[-1,1]	*Uniform*	0	0.58	[-0.9,0.9]
β	$[0,1)$	*Beta*	0.97	0.01	[0.95,0.98]
ϕ	R^+	*Gamma*	0.50	0.20	[0.22,0.87]

We set the standard deviation of σ_s to 0.1 in the pre-Volcker sample and 0.01 in the post-Volcker sample. We made this choice because earlier studies (Primiceri, 2005; Sims and Zha, 2006) found that the variance of shocks was higher in the post-Volcker sample, consistent with the fact that there were two major oil-price shocks in this period.

Table 1.B: Prior distribution for each sample period

Name	Range	Density	Mean	Std. Dev.	90% interval
Pre-Volcker					
λ	R^+	*Gamma*	0.9	0.50	[0.26,1.85]
σ_s	R^+	*Inverse Gamma*	0.1	0.03	[0.06,0.15]
Post-Volcker					
λ	R^+	*Gamma*	1.1	0.50	[0.42,2.02]
σ_s	R^+	*Inverse Gamma*	0.01	0.005	[0.005,0.019]

We restricted the parameters of the policy rule to lie in the indeterminacy region for the pre-Volcker period and the determinacy region, post-Volcker. Those restrictions are consistent with Lubik and Schorfheide (2004) who estimated a NK model, pre and post-Volcker and found that the NK model was best described by an indeterminate equilibrium ion the first sub-period. Our priors for a, λ and μ place the FM model in the indeterminacy region of the parameter space for both sub-samples.

To identify the NK model in the pre-Volcker period, and for the FM model in both sub-periods, we chose a pre-determined price equilibrium. We selected that equilibrium by choosing the forecast error

$$\eta_t^\pi \equiv \pi_t - \mathbb{E}_{t-1}\pi_t$$

as a new fundamental shock and we identified the variance covariance matrix of shocks by setting the covariance of η_t^π with the other fundamental shocks, to zero.

The results of our estimates are reported in Tables 2, 3 and 4. Table 2 reports the logarithm of the marginal data densities and the corresponding posterior model probabilities under the assumption that each model has equal prior probability. These were computed using the modified harmonic mean estimator proposed by Geweke (1999). In Tables 3 and 4 we present parameter estimates for the pre-Volcker period (1954Q3-1979Q2) and the post-Volcker period, (1983Q1-2007).

Table 2: Model comparison

		FM model	NK model
Pre-Volcker (54Q3-79Q2)	Log data density	1023.24	1017.26
	Posterior Model Prob (%)	100	0
Post-Volcker (83Q1-07Q4)	Log data density	1136.22	1121.42
	Posterior Model Prob (%)	100	0

Table 3: Posterior estimates, Pre-Volcker (54Q3-79Q2)

	FM model		NK model	
	Mean	90% probability interval	Mean	90% probability interval
a	3.80	[3.11,4.46]	3.70	[2.91,4.49]
ρ	0.020	[0.012,0.027]	0.017	[0.010,0.023]
η	0.87	[0.83,0.92]	0.76	[0.63,0.89]
$\bar{r}$	0.051	[0.014,0.093]	0.043	[0.002,0.079]
ρ_R	0.94	[0.91,0.97]	0.98	[0.97,0.99]
λ	0.80	[0.22,1.34]	0.45	[0.17,0.73]
μ	0.74	[0.44,1.03]	0.56	[0.28,0.84]
ρ_d	0.76	[0.69,0.83]	0.80	[0.72,0.88]
ρ_s	0.95	[0.92,0.98]	0.78	[0.71,0.86]
σ_R	0.007	[0.006,0.008]	0.008	[0.007,0.009]
σ_d	0.011	[0.009,0.013]	0.011	[0.007,0.014]
σ_s	0.097	[0.059,0.133]	0.059	[0.043,0.073]
σ_ζ	0.003	[0.003,0.004]	0.003	[0.002,0.004]
ρ_{Rd}	0.79	[0.64,0.95]	-0.06	[-0.30,0.17]
ρ_{Rs}	-0.53	[-0.80,-0.26]	0.59	[0.43,0.76]
ρ_{ds}	-0.79	[-0.94,-0.65]	0.11	[-0.22,0.47]
β	n/a	n/a	0.98	[0.97,0.99]
ϕ	n/a	n/a	0.07	[0.04,0.09]

The dynamic properties of the FM model depend on the value of the parameter a. We tried restricting this parameter to be less than 1, a restriction that places the FM model in the determinacy region of the parameter space. We found that the posterior for a model that imposes this restriction was clearly dominated by allowing a to lie in the indeterminacy region. In both the FM and NK cases, we used the approach of Farmer et al. (2015) which allows the econometrician to use standard software packages to estimate indeterminate models.

We see from Table 3, that the estimated parameters of both the FM and NK models, in the first sub-period, are in the region of dynamic indeterminacy. However, the posterior estimates of the policy parameters, $\bar{r}$, ρ_R, λ and μ, are different across the models with substantial differences in λ and ρ_R. Relative to the NK model, the FM model estimates that the monetary authority was more responsive to both changes in the inflation rate from its target (λ) and to changes in the output gap (μ) while the policy regime was less persistent, that is, ρ_R is estimated to be lower.

Table 4 reports the posterior estimates for the post-Volcker period (1983Q1-2007Q4). For this sample period, the FM estimates place the model in the region of dynamic indeterminacy. In contrast, the posterior means of the NK model satisfy the Taylor Principle, thus guaranteeing that the equilibrium of NK model is locally unique.

Table 4: Posterior estimates, Post-Volcker (83Q1-07Q4)

	FM model		NK model	
	Mean	90% probability interval	Mean	90% probability interval
a	4.23	[3.46,4.99]	3.62	[2.87,4.35]
ρ	0.020	[0.012,0.028]	0.023	[0.016,0.029]
η	0.93	[0.88,0.99]	0.93	[0.89,0.98]
$\bar{r}$	0.045	[0.024,0.064]	0.008	[0.001,0.016]
ρ_R	0.75	[0.63,0.88]	0.93	[0.89,0.97]
λ	0.50	[0.17,0.80]	1.39	[1.04,1.70]
μ	0.85	[0.52,1.18]	0.64	[0.34,0.92]
ρ_d	0.78	[0.71,0.85]	0.63	[0.55,0.71]
ρ_s	0.90	[0.84,0.97]	0.94	[0.91,0.98]
σ_R	0.004	[0.004,0.005]	0.006	[0.005,0.006]
σ_d	0.008	[0.006,0.009]	0.007	[0.005,0.009]
σ_s	0.022	[0.008,0.038]	0.011	[0.008,0.014]
σ_ζ	0.005	[0.004,0.006]	n/a	n/a
ρ_{Rd}	-0.47	[-0.67,-0.27]	0.27	[0.10,0.45]
ρ_{Rs}	0.88	[0.77,0.99]	0.20	[0.01,0.40]
ρ_{ds}	-0.62	[-0.89,-0.34]	0.70	[0.56,0.85]
β	n/a	n/a	0.97	[0.95,0.99]
ϕ	n/a	n/a	0.26	[0.11,0.41]

Once again, we find differences in the policy parameters $\bar{r}$, and μ and large significant differences in λ, and ρ_R. Also, in line with previous studies **?**, we find that the estimated volatility of the shocks dropped significantly.

In Section VI we provide further insights on the role that these changes played in affecting the long-run relations between inflation rate, output gap and nominal interest rate.

VI. What Changed in 1980?

There is a large literature that asks: Why do the data look different after the Volcker disinflation? At least two answers have been given to that question. One answer, favored by Sims and Zha (2002), is that the primary reason for a change in the behavior of the data before and after the Volcker disinflation is that the variance of the driving shocks was larger in the pre-Volcker period. Primiceri (2005) finds some evidence that policy also changed but his structural VAR is unable to disentangle changes in the policy rule from changes in the private sector equations.

Previous work by Canova and Gambetti (2004) explains the reduction in volatility after 1980 as a consequence of better monetary policy. But when Lubik and Schorfheide (2004) estimate a NK model over two separate sub-periods they find significant difference across regimes, not only in the policy parameters, but also in their estimates of the private sector parameters. That leads to the following question. Can the FM model explain the change in the behavior of the data before and after 1980 in terms of a change only in the policy parameters? To answer that question, we estimated five alternative models. The results are reported in Table 5.

In Model 1, Fully unrestricted, we estimated all the parameters of the FM model separately for the two sub-periods. In Model 2, Policy and shocks, we allowed the variances of the shocks and the parameters of the policy rule to change across sub-periods, but we constrained the parameters of the IS curve to be the same. In Models 3, Shocks only, we allowed only the variances of the shocks to change and in Model 4, we allowed only the Policy Rule parameters to change. Finally, in Model 5, we restricted all of the parameters to be the same in both sub-periods.

Table 5: Model specifications

	Log data density	Posterior model prob
Fully unrestricted	2159.48	-
Policy and shocks	2159.39	47.7%
Shocks only	2141.56	0%
Policy only	2121.42	0%
Fully restricted	2113.25	0%

The results in Table 5 indicate that the specification in which policy parameters and shocks are allowed to differ explains the data almost as well as the fully unrestricted model specification. But as soon as we restrict either the policy parameters or the shocks to be the same, the explanatory power of the FM model drops substantially. With the exception of Model 2, Policy and shocks, all of the restrictions are clearly rejected.

Our finding is line with the debate on whether the Great Moderation results from either "good policy" or "good luck" and is consistent with the reduced form findings of Primiceri (2005). Our results demonstrate that, conditional on the FM model, the Great Moderation was a combination of both a policy change and "good luck'.

Our results also demonstrate that the conduct of monetary policy affected the long-run relationship between inflation rate and output gap while leaving unchanged our estimate of the Fisher equation. In Table 6.1 and Table 6.2 we report our estimates from Model 2, Policy and Shocks.

Table 6.1: Specification "Policy and Shocks", restricted parameters

	Mean	90% probability interval
a	4.22	[3.58,4.88]
ρ	0.021	[0.013,0.028]
η	0.89	[0.85,0.93]
ρ_d	0.76	[0.71,0.82]
ρ_s	0.95	[0.92,0.98]

Table 6.2: Specification "Policy and Shocks", unrestricted parameters

	pre-Volcker		post-Volcker	
	Mean	90% probability interval	Mean	90% probability interval
$\bar{r}$	0.054	[0.019,0.098]	0.048	[0.026,0.073]
ρ_R	0.98	[0.96,0.99]	0.68	[0.56,0.80]
λ	0.76	[0.19,1.27]	0.39	[0.15,0.62]
μ	0.75	[0.43,1.05]	0.93	[0.60,1.25]
σ_R	0.007	[0.006,0.008]	0.005	[0.004,0.005]
σ_d	0.012	[0.009,0.014]	0.008	[0.006,0.009]
σ_s	0.11	[0.07,0.16]	0.013	[0.008,0.019]
σ_ζ	0.004	[0.003,0.005]	0.006	[0.005,0.006]
ρ_{Rd}	0.77	[0.61,0.94]	-0.44	[-0.64,-0.24]
ρ_{Rs}	-0.57	[-0.83,-0.33]	0.89	[0.77,0.99]
ρ_{ds}	-0.77	[-0.92,-0.64]	-0.52	[-0.84,-0.23]

From these estimates, we can back out the co-integrating equations using the steady state relationships,

$$\bar{\pi} = \frac{(\bar{r} - \rho)}{(1 - \lambda)} + \frac{\mu}{(1 - \lambda)}\bar{y}, \tag{20}$$

$$\bar{R} = \rho + \bar{\pi}. \tag{21}$$

Although our estimates of the Fisher equation in (21) are unchanged, the long-run relationship between the inflation rate and the output gap in (20) varies substantially across regimes. This variation in the implied co-integrating equations is caused by a change in the policy rule pre and post-Volcker. The implied co-integrating equations for the first sub-sample are,

$$\bar{\pi} = 13.7\% + 3.1 * \bar{y}, \tag{22}$$

and for the second,

$$\bar{\pi} = 4.4\% + 1.5 * \bar{y}. \tag{23}$$

These estimates imply that the long-run inflation rate, conditional on a zero output gap, dropped from 13.7% to 4.4%. There is no reason in the FM model for the output-gap to be zero. Instead, the Fed chooses, in every period, if a shock to demand or supply should feed into higher expected inflation or into a higher output-gap. Our estimates imply that the Fed chose to tolerate higher inflation variability, and lower output-gap movements, in the post-Volcker regime, for given shocks to demand and supply.

Why was the post Volcker regime relatively benign? It was not just good policy. The post-Volcker period, leading up to the Great Recession, was associated with fewer large shocks and with no large negative supply shocks of the same order of magnitude as the oil price shocks of 1973 and 1978. If the economy had been hit with negative shocks of that magnitude, our estimates of the co-integrating relationship in this period imply that the outcome would have been a recession of three times the magnitude as in the pre-Volcker regime. Arthur Burns, Chair of the Fed from 1970 to 1978, accepted a big increase in expected inflation following the 1973 oil-price shock. If the oil price shock had hit in 1983, the outcome, instead, would have been a much larger recession.

VII. Conclusions

The FM model gives a very different explanation of the relationship between inflation, the output gap and the federal funds rate from the conventional NK approach. It is a model where demand and supply shocks may have permanent effects on employment and inflation. Our empirical findings demonstrate that this model fits the data better than the NK alternative. The improved empirical performance of this model stems from its ability to account for persistent movements in the data.

In the FM model, beliefs about nominal income growth are fundamentals of the economy. Beliefs select the equilibrium that prevails in the long-run and monetary policy chooses to allocate shocks to permanent changes in inflation expectations or permanent deviations of output from its trend growth path.

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Appendix A: The Reduced Forms of the NK and FM Models

In Appendix A we find solutions to simplified versions of the two models and we show how they are different from each other. To find closed form solutions, we set $\rho = 0$, $\eta = 0$, $a = 1$, $\bar{r} = 0$ and $\rho_R = 0$. These simplifications allow us to solve the models by hand using a Jordan decomposition. For more general parameter values we rely on numerical solutions that we compute using Christopher Sim's code, GENSYS Sims (2001).

A1. Solving the NK Model

Consider the following stripped down version of the NK model

$$\begin{aligned} y_t &= E_t(y_{t+1}) - (R_t - E_t(\pi_{t+1})) \\ R_t &= \lambda \pi_t + \mu y_y + z_{R,t} \\ \pi_t &= \beta E_{t-1}(\pi_{t+1}) + \phi y_t \\ \eta_{1,t} &= y_t - E_{t-1}(y_t) \\ \eta_{2,t} &= \pi_t - E_{t-1}(\pi_t) \end{aligned}$$

The model can be written in the following matrix form

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi z_t + \Pi \eta_t, \tag{A1}$$

where $X_t \equiv (y_t, \pi_t, E_t(y_{t+1}), E_t(\pi_{t+1}))'$, $\varepsilon_t = (z_{R,t})$ and $\eta_t = (\eta_{1,t}, \eta_{2,t})'$.

Defining the matrix $\Gamma_1^* \equiv \Gamma_0^{-1}\Gamma_1$ we may rewrite this equation,

$$X_t = \Gamma_1^* X_{t-1} + \Psi^* \varepsilon_t + \Pi^* \eta_t. \tag{A2}$$

The existence of a unique bounded solution to Equation (A2) requires that two roots of the matrix Γ_1^* are outside the unit circle. This condition is satisfied when the following generalized form of the Taylor Pricipal holds,

$$\left| \lambda + \frac{1-\beta}{\phi}\mu \right| > 1.$$

In this case, the reduced form is an equation,

$$X_t = G^{NK} X_{t-1} + H^{NK} z_t \tag{A3}$$

where H^{NK} is a 5×1 vector of coefficients and G^{NK} is a 5×5 matrix of zeros.

When the Taylor Principal breaks down, one or more elements of the vector of non-fundamental shocks, η_t, can be reclassified as fundamental. In that case, the reduced form can be represented as

$$X_t = G^{NK} X_{t-1} + H^{NK} \begin{bmatrix} z_t \\ \eta_{1,t} \end{bmatrix} \tag{A4}$$

where H^{NK} is a 5×2 vector of coefficients and G^{NK} is a 5×5 matrix of rank 4.

A2. Solving the FM model

The equivalent stripped-down version of the FM model can be written as,

$$\begin{aligned}
y_t &= \mathbb{E}_t[y_{t+1}] - (R_t - \mathbb{E}_t[\pi_{t+1}])\,, \\
R_t &= \lambda \pi_t + \mu y_t + z_{R,t}, \\
\pi_t &= \mathbb{E}_t[\pi_{t+1}] + (\mathbb{E}_t[y_{t+1}] - y_t) - (y_t - y_{t-1})\,. \\
\eta_{1,t} &= y_t - E_{t-1}(y_t) \\
\eta_{2,t} &= \pi_t - E_{t-1}(\pi_t)
\end{aligned}$$

For our parametrization this system is indeterminate and the reduced form is represented by the system

$$X_t = G^{FM} X_{t-1} + H^{FM} \begin{bmatrix} z_t \\ \eta_{1,t} \end{bmatrix} \tag{A5}$$

where H^{FM} is a 5×2 vector of coefficients and G^{FM} is a 5×5 matrix of rank 4. We show in an unpublished appendix, available from the authors, that

$$G^{FM} = \begin{bmatrix} 0 & -\frac{\mu}{\lambda-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{\mu}{\lambda-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{\mu}{\lambda-1} & 0 & 0 & 0 \end{bmatrix}, \qquad H^{FM} = \frac{1}{1+\mu+\phi\lambda} \begin{bmatrix} 1 \\ -1 \\ -\phi \\ 0 \\ 0 \end{bmatrix}. \tag{A6}$$

Note that matrix G^{FM} has a unit entry on the main diagonal of row 2 and zeros everywhere else on that row. This fact implies that G^{FM} has a unit root.

Appendix B: Dynamic Properties for generalized IS curve

We now show that the dynamic properties of the FM model depend not only on the parameters of the monetary policy reaction function but importantly also on the parameter of relative risk aversion a. To simplify the notation, we considering the case of $\rho_R = 0$ and proceed to solve the model as in Appendix A. The roots of the system are $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = 1$ and

$$\lambda_{4,5} = \frac{-(\lambda - \mu - a + 1) \pm \sqrt{(\lambda - \mu - a + 1)^2 + 4\lambda(a-1)}}{2(a-1)}.$$

Given the posterior mean of the parameter $\lambda = 0.92$ and $\mu = 0.99$, we focus on the approximated roots for $(\lambda - \mu) = 0$. Thus, we obtain

$$\lambda_{4,5} = \frac{(a-1) \pm \sqrt{(-a+1)^2 + 4\lambda(a-1)}}{2(a-1)} = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{4\lambda}{(a-1)}}.$$

We first show that the eigenvalue $\lambda_4 = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\lambda}{(a-1)}}$ is always unstable for realistic values of the parameter λ and a. If $(a-1) > 0$, then $\lambda_4 > 1$. If $(a-1) < 0$, then $0 < \lambda_4 < 1$ if and only if $4\lambda < (1-a)$ or equivalently $a < 1 - 4\lambda$. For realistic values of the parameter λ, this is never the case, implying that λ_4 is always an unstable root of the model.

Given that the FM model has two forward-looking variables and that $\lambda_4 > 1$, the model is dynamically determinate if $\lambda_5 = \left[\frac{1}{2} - \frac{1}{2}\sqrt{1 + \frac{4\lambda}{(a-1)}}\right] < -1$. Simplifying, this condition can be written as

$$a < 1 + \frac{\lambda}{2}.$$

The posterior means reported in Table 3 and 4 for both the pre- and post-Volcker period indicate that this condition is violated, and that the dynamic properties of the FM model crucially depend on the value of the parameter a.