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Resource Management with Stochastic Recharge And Environmental Threats

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Abstract

Exploitation diminishes the capacity of renewable resources to withstand environmental stress, increasing their vulnerability to extreme conditions that may trigger abrupt changes. The onset of such events depends on the coincidence of extreme environmental conditions (environmental threat) and the resource state (determining its resilience). When the former is uncertain and the latter evolves stochastically, the uncertainty regarding the event occurrence is the result of the combined effect of these two uncertain components. We analyzed optimal resource management in such a setting. Existence of an optimal stationary policy is established and long run properties are characterized. A numerical illustration based on actual data is presented.

Keywords: Stochastic stock dynamics, event uncertainty, Markov decision process, optimal stationary policy.

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1 Introduction

We study management of renewable resources with a stochastic state evolution and environmental uncertainty regarding the occurrence of an abrupt catastrophic event. The effects on management policies of these two uncertain processes are highly intertwined, as the vulnerability of a resource to (uncertain) environmental stress depends critically on its (stochastic) state. Admittedly, numerous uncertain elements prevail in any given resource situation and the literature addresses many of them (see Pindyck 2007, for a survey). But the combined effect of stochastic state evolution and uncertain abrupt events has not been addressed so far.

The economic literature on natural resources with stochastic state dynamics (e.g., Burt 1964, Reed 1974, Pindyck 1984, 2002, Knapp and Olson 1995, Costello et al. 2001, Singh et al. 2006, Mitra and Roy 2006, Wirl 2007, and references they cite) mostly ignores uncertain catastrophic events such as abrupt regime shift or ecological collapse.[1] The sudden occurrence of such events is related to nonlinear phenomena such as positive feedbacks, hysteresis and the presence of uncertain thresholds that are prevalent in environmental processes (Dasgupta and Mäler 2003, Brock and Starrett 2003). Examples include pollution-induced catastrophes (Cropper 1976, Clarke and Reed 1994, Aronsson et al. 1998, Tsur and Zemel 1998), a sudden collapse of an ecosystem or of animal and plant populations (Reed 1989, Tsur and Zemel 1994, Brock and Xepapadeas 2003), destruction of coastal aquifers due to seawater intrusion (Tsur and Zemel 1995, 2004), phosphorus loading into lakes inducing an

[1]Some of these works incorporate thresholds, e.g., project investment thresholds in Pindyck (2002), extinction thresholds in Mitra and Roy (2006) and temperature thresholds in Wirl (2007), but these thresholds are deterministic and the uncertainty emanates only from the stochastic stock dynamics.

irreversible transition from oligotrophic (clear) state into a eutrophic (turbid) state (Harper 1992, Carpenter et al. 1999, Mäler 2000), and global warming induced catastrophes (Tsur and Zemel 1996, 2009, Broecker 1997, Mastrandrea and Schneider 2001, Alley et al. 2003, Nævdal 2006, Haurie and Moresino 2006, Roe and Baker 2007, Stern 2007, Bahn et al. 2008, Weitzman 2009).[2] This literature strain assumes a deterministic evolution of the resource state.

The most pronounced effect on resource management policies of the presence of a catastrophic threat shows up in the discount factor, which becomes policy-dependent, i.e., endogenous. Implications of this property for climate policies under threats of global warming induced catastrophes have recently been studied (see, e.g., Tsur and Zemel 2008, 2009) in a deterministic resource evolution framework. Here we consider stochastic state dynamics in a general renewable resource situation. The endogeneity of the discount factor requires extending some properties of Markov decision processes that are known to hold under constant discounting (see, e.g., Puterman 2005).

The resource setup, with the stochastic state dynamics and the environmental threat, is formulated in Section 2. Section 3 formulates the management problem. Existence of an optimal stationary policy under policy-dependent discount factor is established in Section 4. Long-run (steady state) behavior under the optimal policy is characterized in Section 5. A numerical illustration, based on actual data, is presented in Section 6. Section 7 concludes and the appendix describes the algorithm used to calculate the optimal policy, value and steady state distribution of the numerical example.

[2]The abrupt change may be desirable, as in Bahn et al. (2008) who consider two such events: the resolution of uncertainty regarding climate sensitivity and technological breakthrough regarding a carbon-free energy source.

2 Resource setup

We formulate the rules governing the evolution of the resource state under the uncertain environmental event.

2.1 States, actions and recharge

The state of the resource system at time t is denoted $S_t = (S_t^1, S_t^2, ..., S_t^M)'$, where S_t^m is the m'th stock, $m = 1, 2, ..., M$. The resource evolves in time according to

$$S_{t+1} = S_t + R(S_t) + X_t - g_t, \ t = 1, 2, \ldots, \tag{2.1}$$

where $R(S_t) = (R^1(S_t), R^2(S_t), ..., R^M(S_t))'$, $X_t = (X_t^1, X_t^2, ..., X_t^M)'$ and $g_t = (g_t^1, g_t^2, ..., g_t^M)'$ are M-dimensional vectors representing deterministic recharge, stochastic recharge and exploitation (harvest, extraction) rates, respectively. We consider discrete (finite or countable) state, recharge and action spaces, denoted $\mathcal{S}$, $\mathcal{X}$ and $\mathcal{A}$, respectively. Thus, $\mathcal{S} = \{s_1, s_2, ..., s_{n_s}\}$, where $s_j \in \mathbb{R}^M$, $j = 1, 2, ..., n_s$ and n_s (possibly infinite) is the number of states.

Let $\mathcal{A}^m(s)$ consists of stock m's actions (exploitation rates) feasible at state $s \in \mathcal{S}$ and let $\mathcal{A}(s) = \mathcal{A}^1(s) \times \mathcal{A}^2(s) \times \cdots \times \mathcal{A}^M(s)$. The admissible action space is $\mathcal{A} = \bigcup_{s \in \mathcal{S}} \mathcal{A}(s) = \{a_1, a_2, ..., a_{n_a}\}$, where $a_j \in \mathbb{R}^M$ and n_a is the number of actions (finite or countable). An action $g_t = (g_t^1, g_t^2, ..., g_t^M)'$ corresponds to exploiting (harvesting, extracting) source m at the rate g_t^m, $m = 1, 2, ..., M$, during time period t. The action is feasible if $g_t \in \mathcal{A}(S_t)$.

In a similar manner we let $\mathcal{X}^m(s)$ represent the support of stock m's recharge distribution at state $s \in \mathcal{S}$ and define $\mathcal{X}(s) = \mathcal{X}^1(s) \times \mathcal{X}^2(s) \times \cdots \times \mathcal{X}^m(s)$. The admissible support is $\mathcal{X} = \bigcup_{s \in \mathcal{S}} \mathcal{X}(s) = \{x_1, x_2, ..., x_{n_x}\}$, containing n_x (possibly infinite) feasible recharge vectors $x_j \in \mathbb{R}_+^M$. The recharge probability at time t, given $S_t = s$, is denoted $p_{x|s}(\cdot)$, i.e.,

$$p_{x|s}(x) \equiv Pr\{R(S_t) + X_t = x | S_t = s\}. \tag{2.2}$$

2.2 Environmental threat

The resource system is under risk of an abrupt shock (regime shift) with undesirable consequences. The conditions that trigger such events depend on the resource state and exploitation policy and are uncertain due to genuine environmental uncertainty or due to our own lack of complete understanding of the processes that lead to occurrence of the event or both. Let κ denote the catastrophic state of the resource system and let $1 - \lambda(s, a)$ be the hazard probability to end up in κ at time $t + 1$ when occupying state $s \neq \kappa$ and employing action a at time t. Let T denote the time period at which the event occurs. Then,

$$Pr\{T = \tau\} = [1 - \lambda(S_\tau, g_\tau)] \prod_{j=1}^{\tau-1} \lambda(S_j, g_j), \ \tau = 1, 2..., \tag{2.3}$$

where we use the convention that $\prod_{j=1}^{\tau-1} = 1$ for $\tau = 1$.

The event occurrence probability (2.3) represents the environmental uncertainty conditional on the resource state trajectory and exploitation policy. The combined effect of the event uncertainty and the stochastic evolution of the resource state shows up in the resource transition probabilities, specified next.

2.3 Transition probabilities

Let $p(j|i, a)$ represent the probability of occupying state s_j at time $t + 1$ conditional on $S_t = s_i$, $g_t = a$ and $T > t$ (i.e., that the event will not interrupt):

$$p(j|i, a) = Pr\{S_{t+1} = s_j | S_t = s_i, g_t = a, T > t\}.$$

In view of (2.1)-(2.2),

$$p(j|i,a) = p_{x|s_i}(s_j - s_i + a). \tag{2.4}$$

We let P_a represent the $n_s \times n_s$ matrix with $p(j|i,a)$ as the (i,j) element.

Given the the event has not occurred by time $t-1$, the probability during time t of moving from s_i to s_j and of nonoccurrence is

$$\begin{aligned} q(j|i,a) &\equiv Pr\{S_{t+1} = s_j, T > t | S_t = s_i, g_t = a\} \\ &= Pr\{S_{t+1} = s_j | S_t = s_i, g_t = a, T > t\} Pr\{T > t | T > t-1, S_t = s_i, g_t = a\} \\ &= p(j|i,a)\lambda(s_i,a). \end{aligned} \tag{2.5}$$

We denote by Q_a the $n_s \times n_s$ matrix with the (i,j) element given by $q(j|i,a)$.

3 Management policies and welfare

We begin by formulating the instantaneous rewards and payoffs. The decision rules and policies are explained next and subsection 3.3 presents the welfare criterion.

3.1 Rewards and payoffs

If the event does not occur during time period t, while the resource is at state S_t and the action g_t is undertaken, the instantaneous reward $\tilde{b}(S_t, g_t)$ is obtained, whereas if the event occurs the post-event value $v^p(S_t)$ is acquired. The latter represents the present-value, under the optimal post-event policy, of the benefit flow from the occurrence time onwards, discounted to the beginning of the occurrence period. We assume that $\tilde{b}(s,a)$ and $v^p(s)$ are bounded and that the latter is smaller than the pre-event value (defined below), as we consider undesirable events.

With $\beta \in [0,1)$ representing the (constant) discount factor, the payoff is given by

$$\sum_{t=1}^{T-1} \tilde{b}(S_t, g_t)\beta^{t-1} + v^p(S_T)\beta^{T-1}. \tag{3.1}$$

The expectation with respect to the event occurrence time T, noting (2.3), is

$$\begin{aligned}
\sum_{\tau=1}^{\infty} \left(\sum_{t=1}^{\tau-1} \tilde{b}(S_t, g_t)\beta^{t-1} + v^p(S_\tau)\beta^{\tau-1} \right) [1 - \lambda(S_\tau, g_\tau)] \prod_{j=1}^{\tau-1} \lambda(S_j, g_j) = \\
\sum_{\tau=1}^{\infty} \sum_{t=1}^{\tau-1} \tilde{b}(S_t, g_t)\beta^{t-1}[1 - \lambda(S_\tau, g_\tau)] \prod_{j=1}^{\tau-1} \lambda(S_j, g_j) + \\
\sum_{\tau=1}^{\infty} v^p(S_\tau)\beta^{\tau-1}[1 - \lambda(S_\tau, g_\tau)] \prod_{j=1}^{\tau-1} \lambda(S_j, g_j).
\end{aligned} \tag{3.2}$$

By changing the order of summation (permitted when $\tilde{b}$ is bounded), the first term on the right-hand side above is expressed as

$$\sum_{t=1}^{\infty} \tilde{b}(S_t, g_t)\beta^{t-1} \sum_{\tau=t}^{\infty} \left([1 - \lambda(S_\tau, g_\tau)] \prod_{j=1}^{\tau-1} \lambda(S_j, g_j) \right). \tag{3.3}$$

The inner sum above equals

$$\sum_{\tau=t}^{\infty} \left(\prod_{j=1}^{\tau-1} \lambda(S_j, g_j) - \prod_{j=1}^{\tau} \lambda(S_j, g_j) \right) = \prod_{j=1}^{t-1} \lambda(S_j, g_j),$$

which upon substituting back in (3.3) gives

$$\sum_{t=1}^{\infty} \left(\tilde{b}(S_t, g_t) \prod_{j=1}^{t-1} \beta\lambda(S_j, g_j) \right). \tag{3.4}$$

This expression is the present value of the benefit flow $\tilde{b}(S_t, g_t)$ discounted with the policy-dependent discount factor

$$\gamma(t) = \begin{cases} 1 & t = 1 \\ \prod_{j=1}^{t-1} \beta\lambda(S_j, g_j) & t = 2, 3, ... \end{cases} \tag{3.5}$$

The function $\gamma(t)$ is the compound discount factor corresponding to the running (single period) discount factor $\beta\lambda(S_t, g_t)$.

The second term on the right-hand side of (3.2) is expressed as

$$\sum_{t=1}^{\infty} v^p(S_t)[1 - \lambda(S_t, g_t)]\gamma(t). \tag{3.6}$$

Combining (3.4) and (3.6), the expectation of the payoff with respect to event occurrence time T is give by

$$\sum_{t=1}^{\infty} b(S_t, g_t)\gamma(t), \tag{3.7}$$

where

$$b(S_t, g_t) \equiv \tilde{b}(S_t, g_t) + v^p(S_t)[1 - \lambda(S_t, g_t)]. \tag{3.8}$$

The catastrophic environmental threat affects the payoff in two ways: first, by changing the instantaneous benefit from $\tilde{b}(S_t, g_t)$ to $b(S_t, g_t)$; second, by changing the discount factor from the constant β to the state-and-action-dependent discount factor $\beta\lambda(S_t, g_t)$. The latter effect is twofold: first, it decreases the discount factor ($\beta\lambda(s, a) \leq \beta$ since $\lambda(s, a) \leq 1$), thereby inducing less conservation (since the future is discounted more heavily); second, it turns the discount factor endogenous to the exploitation policy. The policy implications of these effects were studied in a deterministic state evolution model of climate change induced catastrophes (e.g., Tsur and Zemel 2008, 2009). Here we consider stochastic state evolution.

3.2 Decision rules and policies

A decision rule $d_t(\cdot)$ determines the action at time t given the available information $\{S_t, S_{t-1}, S_{t-2}...\}, \{g_{t-1}, g_{t-2}, ...\}$. It may be history-dependent or Markovian (depends only on the current state S_t), randomized or deterministic. Consequently, the four types of decision rules are history-dependent and randomized (HR), history-dependent and deterministic (HD), Markovian and randomized (MR), Markovian and deterministic (MD). A policy (or plan) specifies the decision rules for all time periods, $\pi = \{d_1, d_2, ...\}$, and is classified as HR, HD, MR or MD depending on the type of the decision rules $d_t,\ t = 1, 2, ...$ A policy is stationary if the same decision rule is repeated in all time periods, i.e., $d_t(\cdot) = \varphi(\cdot)\ \forall t$. (Thus, a stationary policy is necessarily Markovian.)

The HR class of policies is the widest and contains all other classes as special cases, while the MD class is contained in all other classes. Within the MD class, stationary policies are the simplest, hence are attractive for actual implementations.

3.3 Welfare

Under a Markovian policy $\pi = \{d_1, d_2, ...\}$, with $g_t = d_t(S_t)$, the (random) payoff, noting (3.7), is

$$\sum_{t=1}^{\infty} b(S_t, d(S_t))\gamma(t)$$

and the expected payoff given $S_1 = s$ is

$$v^{\pi}(s) = E^{\pi}\left\{\sum_{t=1}^{\infty} b(S_t, d_t(S_t))\gamma(t)\right\}. \tag{3.9}$$

The welfare (value) function is defined as

$$v^{*}(s) = \sup_{\pi \in \Pi^{\mathrm{HR}}} v^{\pi}(s),\ \ s \in \mathcal{S}. \tag{3.10}$$

4 Optimal policy

The optimal policy π^*, when exists, satisfies $v^{\pi^*}(s) = v^*(s)$ for all $s \in \mathcal{S}$. We denote by $v^{\varphi}(s)$ the value corresponding to the stationary policy $\pi = (\varphi, \varphi, \cdots)$. As stationary MD policies are attractive for implementation purposes, it is of interest to know if the value v^* can be attained by such a policy.

Under a constant discount factor (and some regularity conditions), the answer is in the affirmative (see Puterman 2005, Chapter 6). We show that this property is retained with endogenous (policy-dependent) discount factor and, along the way, characterize the optimal stationary policy. We begin with some definitions and notation.

Recall that without the catastrophic threat, i.e., when the survival probability $\lambda(s,a)=1$ for all $s\in\mathcal{S}$ and $a\in\mathcal{A}$, the discount factor is constant and the optimality equations are

$$v(s_i)=\max_{a_i\in\mathcal{A}(s_i)}\left\{b(s_i,a_i)+\beta\sum_{j=1}^{n_s}p(j|i,a_i)v(s_j)\right\},\ i=1,2,...,n_s,$$

or in matrix notation

$$v=\max_{a\in\mathcal{A}}\left\{b_a+\beta P_a v\right\},$$

where $v=(v(s_1),...,v(s_{n_s}))'$, $a=(a_1,...,a_{n_s})\in\mathcal{A}(s_1)\times\cdots\times\mathcal{A}(s_{n_s})=\mathcal{A}(s)$, $b_a=(b(s_1,a_1),...,b(s_{n_s},a_{n_s}))'$ and P_a is the $n_s\times n_s$ matrix with the (i,j) element given by $p(j|i,a)$. In the presence of environmental threat, the discount factor $\beta\lambda(s_i,a)$ is state-and-action-dependent and the optimality equations become

$$v(s_i)=\max_{a_i\in\mathcal{A}(s_i)}\left\{b(s_i,a_i)+\beta\lambda(s_i,a_i)\sum_{j=1}^{n_s}p(j|i,a_i)v(s_j)\right\},\ i=1,2,...,n_s, \tag{4.1}$$

or in matrix notation

$$v=\max_{a\in\mathcal{A}}\{b_a+\beta Q_a v\}, \tag{4.2}$$

where Q_a is an $n_s\times n_s$ matrix with (i,j) element given by $\lambda(s_i,a)p(j|i,a)$ (the i'th row of Q_a equals $\lambda(s_i,a)$ times the i'th row of P_a).

Let V be the space of bounded functions on $\mathcal{S}$ endowed with the supremum norm $\|v\| = \sup_{s\in\mathcal{S}} v(s)$. Define the mapping $L : V \mapsto V$:

$$L(v)_i = \max_{a_i\in\mathcal{A}(s_i)} \left\{ b(s_i, a_i) + \beta\lambda(s_i, a_i) \sum_{j=1}^{n_s} p(j|i, a_i)v(s_j) \right\}, \ i = 1, 2, ..., n_s,$$

or in matrix notation

$$L(v) = \max_{a\in\mathcal{A}} \{b_a + \beta Q_a v\}. \tag{4.3}$$

The optimality equations can be expressed in terms of L as

$$v(s_i) = L(v)_i, \ i = 1, 2, ..., n_s,$$

or in matrix notation as

$$v = L(v). \tag{4.4}$$

We now establish:

Theorem 4.1. *Suppose that (A1) $0 \le \beta < 1$, (A2) $\mathcal{S}$ is discrete (finite or countable), (A3) $\tilde{b} : \mathcal{S} \times \mathcal{A} \mapsto \mathrm{I\!R}$ and $v^p : \mathcal{S} \mapsto \mathrm{I\!R}$ are bounded and (A4) $\tilde{b}(s_i, a)$ and $\lambda(s_i, a)p(j|i, a)$ are continuous in a, and $\mathcal{A}(s_i)$ is compact for all $s_i, s_j \in \mathcal{S}$. Then:*

(i) the optimal value v^* is the unique fixed point of (4.4)*;*

(ii) a stationary policy φ is optimal if and only if the actions $a_i = \varphi(s_i), \ i = 1, 2, ..., n_s$, realize the maxima in (4.1)*;*

(iii) there exists an optimal, Markovian-Deterministic stationary policy φ^*, i.e., the policy $(\varphi^*, \varphi^*, ...)$ satisfies

$$v^{\varphi^*}(s) = v^*(s) \ \forall s \in \mathcal{S}. \tag{4.5}$$

Proof. Assumptions (A3)-(A4) ensure that the maxima in (4.1) are attained. For a given $v \in V$, let $a_i(v), \ i = 1, 2, ..., n_s$, denote the actions where the maxima in (4.1) are attained. Then, for any $u \in V$ we have

$$L(u)_i \geq \left\{ b(s_i, a_i(v)) + \beta\lambda(s_i, a_i(v)) \sum_{j=1}^{n_s} p(j|i, a_i(v))u_j \right\}, \ i = 1, 2, ..., n_s,$$

which together with

$$L(v)_i = b(s_i, a_i(v)) + \beta\lambda(s_i, a_i(v)) \sum_{j=1}^{n_s} p(j|i, a_i(v))v_j$$

implies

$$L(v)_i - L(u)_i \leq \beta\lambda(s_i, a_i(v)) \sum_{j=1}^{n_s} p(j|i, a_i(v))(v_j - u_j), \ i = 1, 2, ..., n_s. \quad (4.6)$$

Since $\sum_{j=1}^{n_s} p(j|i, a_i(v)) = 1$, we conclude from (4.6) that

$$L(v)_i - L(u)_i \leq \beta\lambda(s_i, a_i(v)) \max_j |v_j - u_j|, \ i = 1, 2, ..., n_s.$$

Since $\beta\lambda(s_i, a_i(v)) \leq \beta < 1$, we can further conclude that

$$\max_i \{L(v)_i - L(u)_i\} \leq \beta \max_j |v_j - u_j|.$$

Interchanging in the above inequality the roles of u and v we obtain

$$\max_i |L(v)_i - L(u)_i| \leq \beta \max_j |v_j - u_j|. \quad (4.7)$$

It follows from (4.7) and (A1) that L is a contraction, implying the existence of a unique fixed point of (4.4). Denote this fixed point by $\tilde{v}$. We next show that $\tilde{v} = v^*$.

Let $a_i^*, \ i = 1, 2, ..., n_s$, be the actions that realize the maxima in (4.1), and define $\varphi^*(s_i) = a_i^*$. Then,

$$\tilde{v}(s_i) = b(s_i, \varphi^*(s_i)) + \beta\lambda(s_i, \varphi^*(s_i)) \sum_{j=1}^{n_s} p(j|i, \varphi^*(s_i))\tilde{v}(s_j), \ s_i \in \mathcal{S}, \quad (4.8)$$

or in vector notation

$$\tilde{v} = b_{\varphi^*} + \beta Q_{\varphi^*}\tilde{v}, \quad (4.9)$$

where $b_{\varphi^*} = (b(s_1, \varphi^*(s_1)), ..., b(s_{n_s}, \varphi^*(s_{n_s})))'$ and Q_{φ^*} is the $n_s \times n_s$ matrix with the (i, j) element given by $\lambda(s_i, \varphi^*(s_i))p(j|i, \varphi^*(s_i))$.

Evaluating (4.8) at time t, with $s_i = S_t$ and $g_t = \varphi^*(S_t)$, gives

$$\begin{aligned} \tilde{v}(S_t) &= b(S_t, \varphi^\star(S_t)) + \beta\lambda(S_t, \varphi^*(S_t)) \sum_{j=1}^{n_s} p(j|S_t, \varphi^\star(S_t))\tilde{v}(s_j) \\ &= b(S_t, \varphi^\star(S_t)) + \beta\lambda(S_t, \varphi^*(S_t)) E_t^{\varphi^*} \tilde{v}(S_{t+1}), \end{aligned} \tag{4.10}$$

where $E_t^{\varphi^*}$ denotes expectation under the $g_t = \varphi^*(S_t)$ decision rule conditional on the information available at time t (which includes S_t). Multiplying (4.10) by $\gamma^{\varphi^*}(t)$, where $\gamma(t)$ is defined in (3.5) under the $g_t = \varphi^*(S_t)$ decision rule, and rearranging gives

$$b(S_t, \varphi^\star(S_t))\gamma^{\varphi^*}(t) = \tilde{v}(S_t)\gamma^{\varphi^*}(t) - \gamma^{\varphi^*}(t+1)E_t^{\varphi^*}\tilde{v}(S_{t+1}). \tag{4.11}$$

Since $\gamma^{\varphi^*}(t+1)$ depends only on information available at time t, the second term on the right hand side of (4.11) can be written as

$$\gamma^{\varphi^*}(t+1)E_t^{\varphi^*}\tilde{v}(S_{t+1}) = E_t^{\varphi^*}\left[\gamma^{\varphi^*}(t+1)\tilde{v}(S_{t+1})\right]$$

and (4.11) is written as

$$b(S_t, \varphi^*(S_t))\gamma^{\varphi^*}(t) = \gamma^{\varphi^*}(t)\tilde{v}(S_t) - E_t^{\varphi^*}\left[\gamma^{\varphi^*}(t+1)\tilde{v}(S_{t+1})\right].$$

Taking the unconditional expectation under the $\varphi^*(\cdot)$ decision rule yields

$$E^{\varphi^*} b(S_t, \varphi^\star(S_t))\gamma^{\varphi^*}(t) = E^{\varphi^*}\gamma^{\varphi^*}(t)\tilde{v}(S_t) - E^{\varphi^*}\gamma^{\varphi^*}(t+1)\tilde{v}(S_{t+1}).$$

Summing over $t = 1, 2, ..., \tau$ gives

$$E^{\varphi^*}\sum_{t=1}^{\tau} b(S_t, \varphi^\star(S_t))\gamma^{\varphi^*}(t) = \tilde{v}(S_1) - E^{\varphi^*}\gamma^{\varphi^*}(\tau+1)\tilde{v}(S_{\tau+1}). \tag{4.12}$$

Since $\gamma^{\varphi^*}(\tau) \to 0$ exponentially (uniformly in the policies), letting $\tau \to \infty$ in (4.12) yields

$$E^{\varphi^*} \sum_{t=1}^{\infty} b(S_t, \varphi^*(S_t))\gamma^{\varphi^*}(t) = \tilde{v}(S_1), \tag{4.13}$$

where we use the property that $s_i \mapsto \tilde{v}(s_i)$, $s_i \in \mathcal{S}$, is a bounded function, namely $\tilde{v}$ is a bounded solution of (4.2), which is guaranteed by (A3).

For an arbitrary policy $\varphi(\cdot)$ we can repeat the above derivation with inequalities rather than equalities, obtaining

$$\tilde{v}(S_t) \geq b(S_t, \varphi(S_t)) + \beta\lambda(S_t, \varphi(S_t)) \sum_{j=1}^{n_s} p(j|S_t, \varphi(S_t))\tilde{v}(s_j)$$

instead of (4.10) and

$$E^{\varphi} \sum_{t=1}^{\infty} b(S_t, \varphi(S_t))\gamma^{\varphi}(t) \leq \tilde{v}(S_1)$$

instead of (4.13). It follows that $\varphi^\star(s)$ is an optimal policy and $\tilde{v}(s) = v^*(s)$, establishing claims (i) and (ii) of the theorem. As indicated above, the only condition for the existence of $\varphi^*(\cdot)$ is that there exists a bounded solution for (4.2), which follows from condition (A3) and claim (i), establishing (iii). $\square$

Puterman (2005, Chapter 6,) presents a variety of algorithms for calculating optimal stationary policies of Markov Decision Process (MDP) problems. In the empirical example of Section 6 we calculate the optimal policy using an algorithm based on Linear Programming (LP), adopted to the present case of a state-dependent discount factor.

5 Long-run behavior

Recalling equations (2.4)-(2.5), $P_{\varphi^*}(i, j) = p(j|i, \varphi^*(s_i))$ gives the probability that the resource system moves from $S_t = s_i$ to $S_{t+1} = s_j$ when the optimal policy $g_t = \varphi^*(s_i)$ is employed, conditional on the event not occurring during period t. The unconditional transition probabilities are $Q_{\varphi^*}(i,j) = \lambda_i^* P_{\varphi^*}(i,j)$, where

$$\lambda_i^* \equiv \lambda(s_i, \varphi^*(s_i)),\ i = 1, 2, ..., n_s. \tag{5.1}$$

The transition matrix P_{φ^*} is aperiodic, thus classifies each state as either recurrent or transient. The recurrent states can be arranged in K irreducible subsets $E_k,\ k = 1, 2, ..., K$.[3] Recurrent, irreducible subsets are absorbing, i.e., once the state process enters E_k it stays there forever. We let n_k represent the number of elements (states) in E_k and denote by P_k the $n_k \times n_k$ submatrix of P_{φ^*} corresponding to the states contained in $E_k,\ k = 1, 2, ..., K$.

We call the state s_i "safe" or "unsafe" depending on whether $\lambda_i^* = 1$ or $\lambda_i^* < 1$, respectively. The subset

$$\mathcal{S}_1 = \{s_i \in \mathcal{S} | \lambda_i^* = 1\} \tag{5.2}$$

contains all "safe" states. ($\mathcal{S}_1$ may well be empty.)

If E_k contains no "unsafe" states, i.e., $E_k \subseteq \mathcal{S}_1$, then entering E_k ensures that the event will never occur. This is so because the probability that the event will occur during period t given $S_t = s_i \in E_k$ is $1 - \lambda_i^* = 0$ for any $s_i \in E_k$ and E_k is absorbing. For recurrent, irreducible sets containing only "safe" states we define the limiting matrix[4]

$$\hat{P}_k = \lim_{\tau \to \infty} P_k^\tau. \tag{5.3}$$

The (i,j) element of $\hat{P}_k$ represents the probability that in the long run the system will occupy state s_j when it starts at state s_i and the optimal policy

[3] The subset $E_k \subset \mathcal{S}$ is closed if $Pr\{S_{t+\tau} = s_j | S_t = s_i, \varphi^*(\cdot)\} = 0$ for any $s_i \in E_k$ and $s_j \notin E_k$, $\tau = 1, 2, ...$ The subset E_k is irreducible if no proper subset of it is closed.

[4] The limit exists since P_k is aperiodic.

is employed for any $s_i, s_j \in E_k$. Clearly, $\hat{P}_k$ satisfies $P_k \hat{P}_k = \hat{P}_k$ (taking one extra step cannot change the limiting behavior), implying that $\hat{P}_k$ has identical rows $\hat{p}_k{}'$, given by the solution of (see Puterman 2005, pp. 591-592)

$$q' = q' P_k \text{ subject to } \sum_{j=1}^{n_k} q_j = 1. \tag{5.4}$$

The n_k-vector $\hat{p}_k$ constitutes the steady-state distribution of the n_k states contained in $E_k \subseteq \mathcal{S}_1$, provided the optimal state process begins at (or enters after a finite time) E_k.

If $E_k \nsubseteq \mathcal{S}_1$ (i.e., E_k contains at least one "unsafe" state s_u, say), then entering E_k implies that the event will (eventually) occur with probability one. This is so because each time the "unsafe" state s_u is visited an occurrence probability of $1 - \lambda_u^* > 0$ is inflicted and (once in E_k) visits to s_u never stops prior to the event occurrence.[5] It follows that the limiting probability of $s_i \in E_k \nsubseteq \mathcal{S}_1$ must vanish and the limiting probability of κ (the occurrence state) is one.

We summarize the above discussion in:

Proposition 5.1. *The optimal state process either initiates at or enters after a finite (transient state) period one of the recurrent, irreducible subsets E_k (provided the event has not occurred during the transient state period).*
(i) If $E_k \subseteq \mathcal{S}_1$, then (a) the long run (steady state) probability of states in E_k is given by $\hat{p}_k$, defined in (5.4), (b) the long run probability of states not in E_k vanish, and (c) the event occurrence probability is zero.

[5]Suppose, without loss of generality, that s_u is the only "unsafe" state in E_k and notice that, unless interrupted by the event, the recurrent state s_u will be visited infinite number of times with probability one. Occurrence may happen on the first visit with probability $1 - \lambda_u^*$ or on the second visit with probability $\lambda_u^*(1 - \lambda_u^*)$ or on the third visit with probability $\lambda_u^{*2}(1 - \lambda_u^*)$ and so on. Summing all possibilities gives the occurrence probability $(1 - \lambda_u^*) \sum_{j=0}^{\infty} (\lambda_u^*)^j = 1$.

(ii) If $E_k \nsubseteq \mathcal{S}_1$, then (a) the long run (steady state) probability of all states in $\mathcal{S}$ vanish, and (b) the long run occurrence probability (the limiting probability of the occurrence state κ) is 1.

6 Empirical illustration

The Kinneret water basin (Lake Kinneret is also known as the Sea of Galilee) is the largest of Israel's natural water sources, providing about 40 percent of the country's annual natural water supply on average. Lake Kinneret is a shallow lake, with maximal and average water depths of 46 m and 25 m, respectively (Gvirtzman 2002, p. 34). Like other shallow lakes (Harper 1992, Mäler 2000), it faces a risk of abrupt ecosystem collapse due to pollution and eutrophication processes (Gvirtzman 2002, pp. 43-55). The risk of such abrupt regime-shift depends on the lake's water head (stock). This property, together with the highly volatile recharge process (Figure 1), render the above framework particularly suitable for studying optimal management policies.

In the next subsection we describe the basin's recharge process and derive its distribution. Subsection 6.2 defines states and actions and subsection 6.3 derives the ensuing transition probabilities. The rewards are specified in subsection 6.4, paying special attention to the catastrophic threat associated with over-exploitation. In subsection 6.5 we apply an algorithm based on Linear Programming (LP) for solving Markov decision Processes (MDPs) and derive the optimal policy and value (the algorithm is briefly described the appendix). Finally, the steady state distribution under the optimal policy is calculated in subsection 6.6.

6.1 Recharge process

Figure 1 presents the Kinneret's net (accounting for evaporation) annual recharge for the period 1932 - 2008.[6] We use the gamma distribution to approximate the recharge distribution, i.e., we assume that the recharge series consists of iid draws from a gamma distribution with parameters α and θ, satisfying

$$\alpha\theta = \text{ Mean(recharge) - Min(recharge) } = 570.38 - 157 = 413.38 \text{ MCMY}$$

and

$$\alpha\theta^2 = \text{ Var(recharge) } = 77333.8,$$

where MCMY stands for million m^3 per year (the mean, min and standard deviation of the recharge series are displayed in Figure 1). We obtain $\alpha = 2.20967$ and $\theta = 187.077$. Figure 2 depicts the empirical distribution of the recharge series (dots) and the gamma distribution with the above (α, θ) parameters.

Figure 1

Figure 2

The support of the recharge distribution is denoted $\mathcal{X} = \{x_1, x_2, ..., x_{n_x}\}$, with $x_1 = 150$ MCMY (the minimal recharge realization – see Figure 1), $x_{n_x} = 1450$ MCMY (approximately the maximal recharge realization) and $x_{\ell+1} - x_\ell = \Delta_x,\ \ell = 1, 2, ..., n_x - 1$. Thus,

$$x_\ell = 150 + (\ell - 1)\Delta_x,\ \ell = 1, 2, ..., n_x, \tag{6.1}$$

[6]The help of Miki Zaide, Avihai Hadad and Amir Givati, of Israel's Water Authority, in making the data available for our use is gratefully acknowledged.

and $p_{x|s}(x_\ell)$ is calculated as

$$p_{x|s}(x_\ell) = \begin{cases} F(x_\ell + \Delta_x/2) & \text{if } \ell = 1 \\ F(x_\ell + \Delta_x/2) - F(x_\ell - \Delta_x/2) & \text{if } 2 \le \ell \le n_x - 1 \\ 1 - F(x_\ell - \Delta_x/2) & \text{if } \ell = n_x \end{cases} \tag{6.2}$$

where $F(\cdot)$ is the gamma distribution specified above (and depicted in Figure 2). Since n_x and Δ_x are related according to $x_{n_x} = x_1 + (n_x - 1)\Delta_x$, setting one parameter determines the other. Setting $\Delta_x = 50$ MCMY implies $n_x = 15$.

6.2 States and actions

The Kinneret water-head ranges between the altitudes 208.8 and 215 meter below sea level (-208.8 m and -215 m, respectively). Above the upper water-head (-208.8 m) the water overflows the lake's edges (flooding is avoided by opening the gates of the Degania dam at the southern outlet of the lake leading into the lower Jordan river). The lower altitude (-215 m) is the minimal water head level at which water can be pumped (due to pumping infrastructure) and is designated as the black line.[7] In between there is the so-called red line – an imaginary water-head level indicating a critical water stock below which the above-mentioned catastrophic risk increases sharply. The red line is set at -213 m.[8]

The water stock corresponding to the black line is normalized at zero and each meter of water-head above the black line is equivalent to 165 - 170 million m^3 (MCM).[9] A water state corresponds to the water stock above the black line, so $s = 0$ when the water-head level is at -215 m, $s = 300$ MCM when the water head is at the red line (-213 m) and $s = \bar{s} = 1000$ when the water-head

[7]The exact minimal water head from which pumping is feasible is -214.87 m and we round it to -215 m.

[8]The red line has been modified in the past in response to pressure to increase pumping during dry years (see Gvirtzman 2002, p. 36).

[9]The range is due to differences in the surface of the lake at different water levels.

level is at -208.8 m. The admissible state set is $\mathcal{S} = \{s_1, s_2, ..., s_{n_s}\}$, where the s_j's are evenly spread apart. Setting $s_{j+1} - s_j \equiv \Delta_s = 50$ MCM gives $n_s = 21$ states.

An action a corresponds to pumping a million m^3 per year (MCMY). The admissible action set is $\mathcal{A} = \{a_1, a_2, ..., a_{n_a}\}$ with $a_1 = 0$, $a_{n_a} = 700$ MCMY (determined by the existing pumping infrastructure) and $a_{j+1} - a_j = \Delta_a,\ j = 1, 2, ..., n_a - 1$. Setting $\Delta_a = 50$ MCMY implies $n_a = 15$.

A time period (a year) in the present case begins at the end of the rainy season (the bulk of the rain in Israel's Mediterranean weather occurs during the months of November through April) while water extraction occurs mostly during the dry season (May - October). It is therefore not feasible to extract more than the water stock available at the beginning of the period, i.e., given the water stock S_t at the beginning of period t, $g_t \leq S_t$. Thus, $\mathcal{A}(S_t) = \{a_k \in \mathcal{A} | a_k \leq S_t\}$. At the end of the dry season, the water stock will reach the level $S_t - g_t \geq 0$ and this level affects the catastrophic hazard, as explained in subsection 6.4.

6.3 Transition probabilities

The transition probabilities, conditional on nonoccurrence, are

$$\begin{aligned} p(j|i, a_k) &= Pr\{S_{t+1} = s_j | S_t = s_i, g_t = a_k\} \\ &= Pr\{R(S_t) + X_t = s_j - s_i + a_k\} \\ &= p_{x|s}(s_j - s_i + a_k\},\ j, i = 1, 2, ..., n_s,\ k = 1, 2, ..., n_a, \quad (6.3) \end{aligned}$$

where $p_{x|s}(\cdot)$ is defined in (6.2).

6.4 Instantaneous benefit

The immediate reward at time t, specified in (3.8), is repeated here for convenience:

$$b(S_t, g_t) = \tilde{b}(S_t, g_t) + v^p(S_t)[1 - \lambda(S_t, g_t)].$$

The first term on the right-hand side is the benefit enjoyed during non-occurrence periods; the second term is the benefit under the interrupting regime-shift, namely the post-event value weighted by the occurrence probability. The former consists of the surplus water users (irrigators, households, industry) derive from the pumped water g_t net of the supply cost (extraction, conveyance, treatment, distribution); the latter stems from the forgone benefit associated with not being able to use the lake for a prolong period of time. We discuss each in turn.

6.4.1 Immediate benefits during non-occurrence periods

Let $D(\cdot)$ denote the inverse demand facing the Kinneret's water, i.e., at a water price $\$D(a)$ per million m^3 (MCM) the water demand is a million m^3 per year (MCMY). Let $C(a)$ represent the cost of supplying a MCMY. The consumer surplus, net of the supply cost, associated with the consumption of a MCMY is

$$\int_0^a D(\xi)d\xi - C(a).$$

Assuming that the derived demand for water is inversely related to the water price, i.e., $D(a) = c_1/a$, and that $C(a) = c_2 a$, the net consumer surplus becomes

$$\tilde{b}(s, a) = c_1 \ln(a) - c_2 a, \tag{6.4}$$

where c_1 is a positive demand parameter and c_2 is the unit cost of water supply.

Assuming further that at a price of $\$0.5 \times 10^6$ per MCM ($\$0.5$ per m^3) the water demand is 600 MCMY implies $c_1 = 300 \times 10^6$. The unit cost of supply is taken at $\$0.2 \times 10^6$ per MCM ($c_2 = 0.2 \times 10^6$).

6.4.2 Post-event value and occurrence probability

We consider the case in which the event (the abrupt regime shift) renders the lake's water unusable for a very long period and take the post-event value v^p to represent the forgone consumer surplus (i.e., the benefit water users could derive had the regime shift been prevented) as well as ecological damages and loss of recreational opportunities. We estimate this forgone value by the present value of constant flow $\tilde{b}(s,a)$ evaluated at $a = 550$ MCMY (which is about the average recharge). Thus, with the discount factor $\beta = 0.9434$ (corresponding to 6% interest rate) and the above specification of $\tilde{b}$,

$$v^p = -\tilde{b}(s, 550)/(1-\beta) \approx -3 \times 10^{10}.$$

The survival probability $\lambda(S_t, g_t)$ equals one if $S_t - g_t$ (the minimal water stock during time period t) does not fall below the critical water stock $s_c = 300$ MCM corresponding to the red line. As soon as the water-head drops below the red line, the survival probability decreases and reaches $\lambda(0) = \lambda_0 \geq 0$ at $s = 0$ (the black line). We use the following specification of the survival probability:

$$\lambda(s,a) = \begin{cases} \lambda_0 + (1-\lambda_0)\exp\{\delta(s-a-s_c)/(s-a)\} & \text{if } s-a < s_c \\ 1 & \text{if } s-a \geq s_c \end{cases} \tag{6.5}$$

where δ is a (positive) shape parameter. Indeed for $a = s$, exploitation brings the water stock to the black line and $\lambda(s,s) = \lambda_0$.

The immediate benefit specializes to

$$b(s,a) = c_1 \ln(a) - c_2 a + v^p(1-\lambda_0)\max\{1-\exp[\delta(s-a-s_c)/(s-a)], 0\}. \tag{6.6}$$

The function specifications and parameter values are summarized in Table 1.

Table 1

6.5 Optimal policy and value

We calculate the optimal policy using an algorithm based on Linear Programming (LP). Appendix A describes the algorithm and its application in the present case. The algorithm provides the optimal policy $\varphi^*(s_i),\ i = 1, 2, ..., n_s$, depicted in Figure 3.

Figure 3

Noting (4.9) and $\tilde{v} = v^*$, the value $v^* = (v^*(s_1), ..., v^*(s_{n_s}))'$ is calculated by

$$v^* = (I - \beta Q_{\varphi^*})^{-1} b_{\varphi^*}, \tag{6.7}$$

where $b_{\varphi^*} = (b(s_1, \varphi^*(s_1)), ..., b(s_{n_s}, \varphi^*(s_{n_s}))'$ and Q_{φ^*} is the $n_s \times n_s$ matrix with $\lambda(s_i, \varphi^*(s_i))p(j|i, \varphi^*(s_i))$ as the (i, j) element. The value is depicted in Figure 4.

Figure 4

6.6 Steady state

From the optimal extraction policy in Figure 3 we conclude that there is one recurrent, irreducible subset $E_1 = \{450, 500, ..., 1000\}$, and all states below 450 MCM are transient. This is so because the optimal extraction policy is such that it is not optimal to intentionally drop the water stock below 300 MCM (the red line) at the end of the dry season, and the minimal recharge (during the rainy season) is 150 MCMY. Thus, at the end of the year the water stock will be at or above 450 MCM. Water stocks (at the end of the rainy season) below 450 can only be encountered initially and for a limited number of periods (until recharge increases the stock), hence are transient.[10]

The λ_j^* data of Figure 3 reveal that E_1 contains only "safe" states ($\lambda_i^* = 1$ for all $s_i \in E_1$). Thus, once the optimal state process enters E_1 the event will never occur (the environmental threat is removed).

The steady state probabilities, characterized in Proposition 5.1 and applied with the above E_1, are depicted in Figure 5. In the long run (steady state), under the optimal policy, the stock never drops below 450 MCM (the red line, below which the environmental threat is activated, is at 300 MCM). This allows pumping at least 150 MCMY without drawing the water head below the red line (recall that the water head at the end of the dry season reaches $S_t - g_t$), thereby providing a buffer against bad draws (dry years).

Figure 5

The average long-run stock and extraction are, respectively,

$$\hat{s} = \sum_{j=1}^{n_s} q_j^* s_j = 834.003 \text{ MCM}$$

and

$$\hat{g} = \sum_{j=1}^{n_s} q_j^* \varphi^*(s_j) = 494.211 \text{ MCMY}.$$

If the recharge were stable at the mean $\bar{x} = 570.38$ MCMY (see Figure 1), the steady-state extraction were set at this rate and this policy could have been maintained at a much lower stock level, e.g., at 300 MCM corresponding to the threshold stock (the red line water-head level). The higher (average)

[10]This state classification can be reached also by applying the procedure described in Puterman (2005, p. 590) on the transition matrix P_{φ^*}.

stock constitutes a buffer that allows mitigating extraction fluctuations, in spite of the stochastically fluctuating recharge, by drawing down the stock during bad (low recharge) years and filling it up during good (high recharge) years. On average, extractions are slightly less than the average recharge (494 MCMY vs. 570 MCMY), while under the steady state distribution the optimal extractions' standard deviation,

$$\sqrt{\sum_{j=1}^{n_s} \hat{q}_j[\varphi^*(s_j) - \hat{g}]^2} = 117.225,$$

is substantially smaller than the recharge process' standard deviation of 278.09 (see Figure 1). The latter owes to the buffer role of the water stock (this effect is similar to, though not the same as, the buffer value proposed by Tsur and Graham-Tomasi 1991).

The large lung-run probability of the full capacity stock (the steady-state probability of $s = 1000$ MCM is about 1/3, implying that, under the optimal policy, in the long run the lake should be filled up every third winter on average) is an outcome of the policy of maintaining a large average stock (as a buffer against a series of dry years). Thus, it pays to let more water flow into the lower Jordan river (by opening the gates of Degania dam at the lake's southern outlet during rainy years) in order to have the buffer stock available during dry years. We note that this property is linked to the particular specifications and parameter values of Table 1, set for illustration purpose only.

7 Concluding comments

Exploitation has diminished the capacity of many renewable resources to endure stress, increasing their vulnerability to extreme environmental conditions that may trigger abrupt changes. The onset of such events depends on the coincidence of extreme environmental conditions and the resource state. Typically, both elements are uncertain and the uncertainty associated with the event occurrence is the result of their combined effect. We analyzed resource management in such a setting.

The environmental threat affects management policies in two ways: first, it changes the immediate benefit flow; second, it turns the discount factor endogenous to the exploitation policy. These effects were studied in a variety of resource management problems under a deterministic state evolution (e.g., Clarke and Reed 1994, Tsur and Zemel 1996, 2008, 2009, Aronsson et al. 1998, Haurie and Moresino 2006). Here they are investigated in resource situations involving stochastic state evolution. Existence of an optimal stationary policy is established and long run properties are characterized. A numerical illustration based on actual data is presented.

With some modifications, the framework developed here can be extended to accommodate models that combine resource exploitation and economic growth, such as integrated assessment models of climate change. The evolution of the various state variables in such models is all but stochastic and threats of global warming induced catastrophes have become increasingly alarming (see Nordhaus 2008, Chapter 7). The present framework can be used to incorporate both types of uncertainty in a coherent (non ad hoc) fashion.

A Appendix: The LP algorithm for calculating optimal policies of MDPs

Puterman (2005, Chapter 6) presents a variety of algorithms for calculating optimal policies of Markov decision processes (MDPs). We use the algorithm based on Linear Programming (LP), adopted to the present case of a state-dependent discount factor. We briefly describe the algorithm and its application.

A.1 The LP approach for solving MDPs

The algorithm is based on the following property:

Proposition A.1. *If $v \in V$ satisfies $v \geq L(v)$, then $v \geq v^*$.*

Proof. The mapping L, defined in (4.3), is monotonic, i.e., for $v, u \in V$, $v \geq u$ implies $L(v) \geq L(u)$. This property follows from $\beta \geq 0$ and $Q_a(i,j) \geq 0 \ \forall (i,j)$. Thus, $v \geq L(v)$ implies $L(v) \geq L(L(v)) \equiv L^2(v)$, hence $v \geq L(v)$ implies $v \geq L^2(v)$. Repeating this reasoning, we find that $v \geq L(V)$ implies $v \geq L^k(v)$ for $k = 1, 2, ...$ Letting $k \to \infty$, recalling that L is a contraction and v^* is the unique fixed point of $v = L(v)$ (Theorem 4.1), establishes the result. $\square$

It follows that the inequality $v \geq L(v)$, or in component notation

$$v_i \geq b(s_i, a_k) + \beta\lambda(s_i, a_k) \sum_j p(j|i, a_k) v_j \ \forall a_k \in \mathcal{A}(s_i), \ i = 1, 2, ..., n_s,$$

can at best hold as equality, in which case $v = v^*$. This suggests the following (primal) Linear Programming (LP) problem for finding v^*:
Set $\alpha_j > 0, \ j = 1, 2, ..., n_s$, satisfying $\sum_j \alpha_j = 1$ (any positive α_j will do but

the requirement that they sum to one allows a probability interpretation) and find (unconstrained) $v_j,\ j = 1, 2, ..., n_s$, in order to minimize

$$\sum_{j=1}^{n_s} \alpha_j v_j$$

subject to

$$v_i - \beta\lambda(s_i, a_k) \sum_{j=1}^{n_s} p(j|i, a_k) v_j \geq b(s_i, a_k)\ \forall a_k \in \mathcal{A}(s_i),\ i = 1, 2, ..., n_s.$$

This LP problem has n_s unknowns (columns) and $\sum_{i=1}^{n_s} n_{a_i}$ constraints (rows), where n_{a_i} is the number of actions in $\mathcal{A}(s_i)$.

The dual to the above LP problem is formulated as follows:

Find $x(s_i, a_k) \geq 0,\ i = 1, 2, ..., n_s,\ a_k \in \mathcal{A}(s_i)$, in order to maximize

$$\sum_{i=1}^{n_s} \sum_{a_k \in \mathcal{A}(s_i)} b(s_i, a_k) x(s_i, a_k) \tag{A.1}$$

subject to

$$\sum_{a_k \in \mathcal{A}(s_j)} x(s_j, a_k) - \sum_{i=1}^{n_s} \sum_{a_k \in \mathcal{A}(s_i)} \beta\lambda(s_i, a_k) p(j|i, a_k) x(s_i, a_k) = \alpha_j,\ j = 1, 2, ..., n_s. \tag{A.2}$$

The dual LP has $\sum_{i=1}^{n_s} n_{a_i}$ unknowns (columns) and n_s constraints (rows). The number of constraints is smaller than that of the primal LP problem, which renders the dual LP more tractable. Properties of the dual LP problem, including a verification that a basic solution exists, are discussed in Puterman (2005, pp. 223-231).

Let $x^*(s_i, a_k),\ i = 1, 2, ..., n_s,\ k = 1, 2, ..., n_{a_i}$, denote the solution of the dual LP. Since the dual LP has n_s constraints, only n_s out of the $\sum_{i=1}^{n_s} n_{a_i}$ elements of x^* are positive. Moreover, for any state s_i only one $x^*(s_i, a_k) > 0$. The optimal (Markov-deterministic) stationary policy is specified as

$$\varphi^*(s_i) = \sum_{a_k \in \mathcal{A}(s_i)} \mathbf{1}(x^*(s_i, a_k) > 0) a_k,\ i = 1, 2, ..., n_s, \tag{A.3}$$

where $\mathbf{1}(\cdot)$ assumes the values 1 or 0 when its argument is true or false, respectively.

A.2 LP specification in the present case

Let $D(i,k) = 1$ or 0 as $s_i \geq a_k$ or $s_i < a_k$, respectively. Thus, $D(i,k) = 1$ if the action a_k is feasible at s_i and $D(i,k) = 0$ otherwise (see discussion in subsection 6.2). Let B be the $n_s \times n_a$ matrix with the i,k element given by $b(s_i, a_k)D(i,k)$, where $b(s,a)$ is defined in (6.6). The LP objective (A.1) can be rendered as

$$\sum_{i=1}^{n_s} \sum_{k=1}^{n_a} B(i,k)x(i,k). \tag{A.4}$$

Similarly, let $\tilde{p}(j|i,a_k) = \lambda(s_i,a_k)p(j|i,a_k)D(i,k)$, where $p(j|i,a_k)$ is defined in (6.3). Then

$$\sum_{i=1}^{n_s} \sum_{a_k \in \mathcal{A}(s_i)} p(j|i,a_k)x(i,k) = \sum_{i=1}^{n_s} \sum_{k=1}^{n_a} \tilde{p}(j|i,a_k)x(i,k)$$

and the dual LP constraints (A.2) can be expressed as

$$\sum_{i=1}^{n_s} \sum_{k=1}^{n_a} D(i,k)x(i,k) - \beta \sum_{i=1}^{n_s} \sum_{k=1}^{n_a} \tilde{p}(j|i,k)x(i,k) = 1/n_s,\ j = 1,2,...,n_s, \tag{A.5}$$

where we set $\alpha_j = 1/n_s,\ j = 1,2,...,n_s$.

The LP problem then is to find $x(i,k) \geq 0,\ i = 1,2,...,n_s,\ k = 1,2,...,n_a$, in order to maximize (A.4) subject to (A.5).

Table 1: Specifications and parameter values

Function	Form	Description
$\tilde{b}(s,a)$	$c_1 \log(a) - c_2 a$	Reward under no occurrence
$v^p(s)$	Constant	Post-event value
$\lambda(s,a)$	$\min\left\{1, \lambda_0 + (1-\lambda_0)e^{\delta(s-a-s_c)/(s-a)}\right\}$	Survival probability
Parameter	**Value**	**Description**
β	0.9434	Discount factor =1/(1+0.06)
α	2.20967	Recharge dist. parameter
θ	187.077	Recharge dist. parameter
Δ_s	50 MCM	Diff between consecutive states
n_s	21	Number of admissible states
Δ_a	50 MCMY	Diff between consecutive actions
n_a	15	Number of admissible actions
Δ_x	50 MCMY	Diffe between consecutive recharge
n_x	26	Number of recharge points
c_1	300×10^6	Demand parameter
c_2	0.2×10^6	Unit supply cost
v^p	-3×10^{10}	Forgone benefit due to occurrence
s_c	300 MCM	Critical stock (at red line)
λ_0	0.5	Survival prob at $s = 0$ (black line)
δ	0.2	Hazard parameter

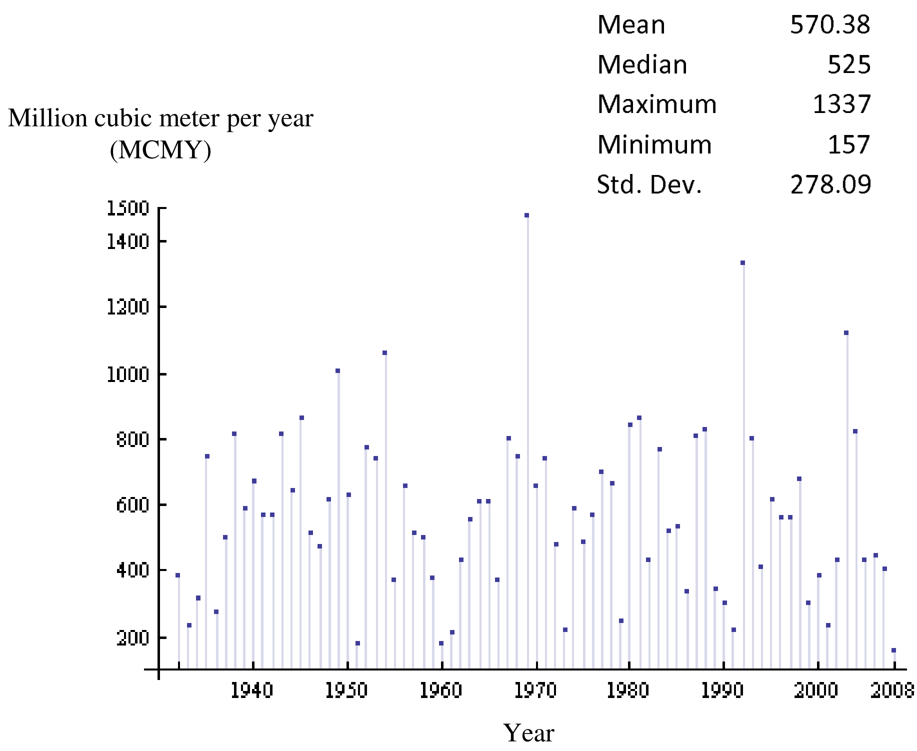


Figure 1: Lake Kinneret's recharge series during 1932 - 2008. The descriptive statistics are calculated for the 1980 - 2008 data.

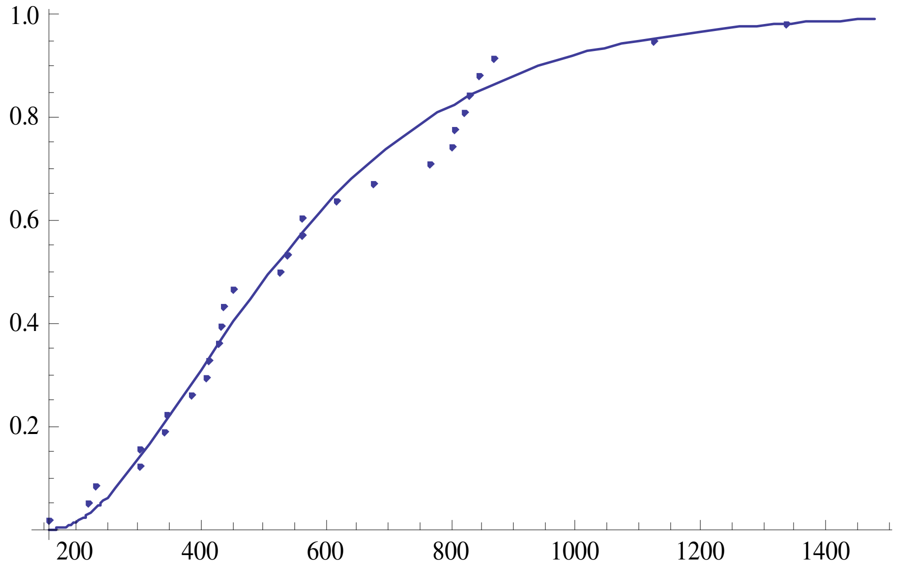
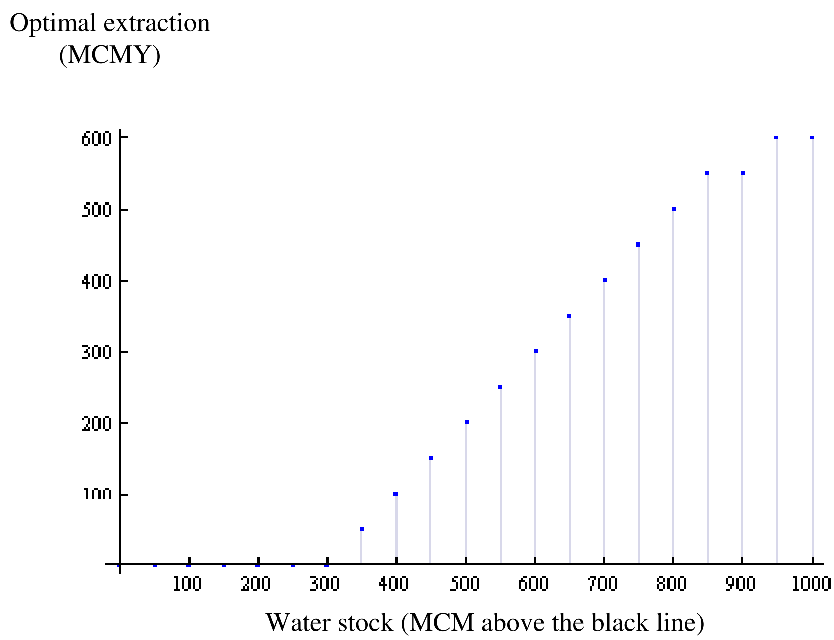
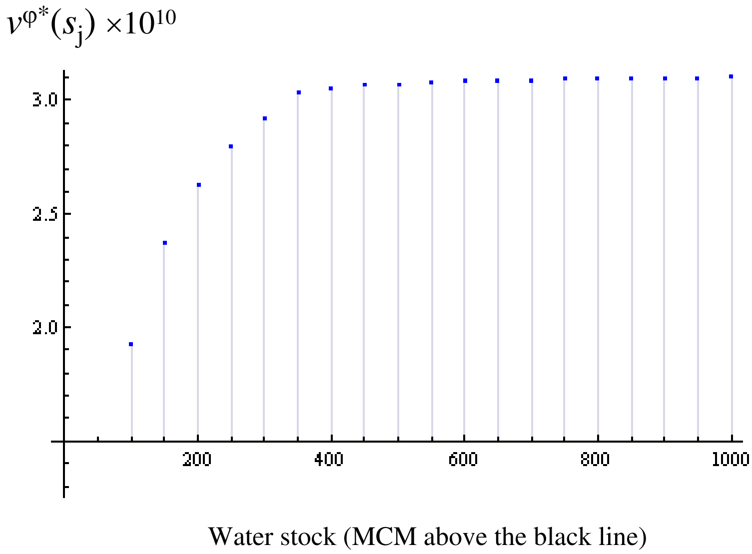


Figure 2: The gamma distribution with parameters $\alpha = 2.20967$ and $\theta = 187.077$ (solid) and the empirical distribution (dots) of the Kinneret's recharge series for the period 1980 - 2008.



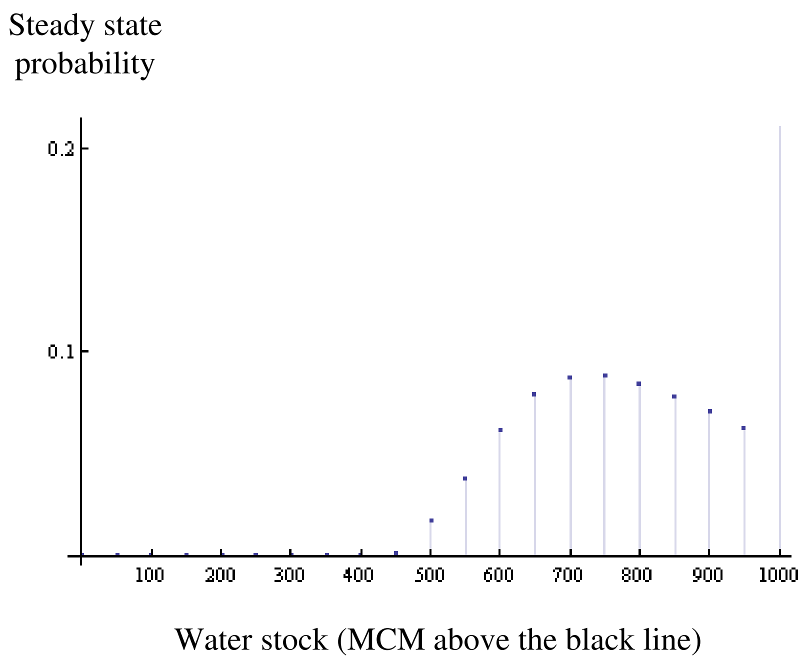
s_j	$\varphi^*(s_j)$	λ^*_j
0	0	0.5
50	0	0.68394
100	0	0.83516
150	0	0.909365
200	0	0.952419
250	0	0.980395
300	0	1
350	50	1
400	100	1
450	150	1
500	200	1
550	250	1
600	300	1
650	350	1
700	400	1
750	450	1
800	500	1
850	550	1
900	550	1
950	600	1
1000	600	1

Figure 3: The optimal stationary Markov extraction policy $\varphi^*(s)$ (MCMY) for $s = 0, 50, 100, ..., 1000$. The data are reported to the right of the figure and contain also the survival probabilities λ_j^*.



s_j	$v^{\varphi^*}(s_j)$
0	-0.0699987
50	1.02578
100	1.92885
150	2.37428
200	2.63255
250	2.8008
300	2.91903
350	3.03599
400	3.05549
450	3.06655
500	3.07413
550	3.0798
600	3.08425
650	3.08786
700	3.09085
750	3.09338
800	3.09553
850	3.09739
900	3.09919
950	3.1008
1000	3.10226

Figure 4: The value $v^{\varphi^*}(s)$ ($\times 10^{10}$ \$) for $s = 0, 50, 100, ..., 1000$ MCM.



s_j	Steady state prob.
0	0
50	0
100	0
150	0
200	0
250	0
300	0
350	0
400	0
450	0.00116072
500	0.0173963
550	0.038434
600	0.0621503
650	0.0791231
700	0.0868794
750	0.0878662
800	0.0843703
850	0.0781758
900	0.0705778
950	0.0624706
1000	0.331396

Figure 5: Long run (steady state) probabilities.

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