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Irrigation Production Functions with Water-Capital Substitution

by

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Abstract

The dynamics of biomass growth implies that the yield of irrigated crops depends, in addition to the total amount of water applied, on irrigation scheduling during the growing period. Advanced irrigation technologies relax constraints on irrigation rates and timing, allowing to better adjust irrigation scheduling to the varying needs of the plants along the growing period. Irrigation production functions, then, should include capital (or expenditures on irrigation equipment) in addition to aggregate water. We derive such functions and study their water-capital substitution properties. Implications for water demand and adoption of irrigation technologies are investigated. An empirical application confirms these properties.

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1 Introduction

As the demand for water for industrial and urban use soars, farmers are pressed to use irrigation water more effectively, either by switching to crops that require less water or water of lower quality, or by improving the irrigation technology. Against this need, the literature suggests a limited capacity to improve the yield derived from a given amount of water because the assumed forms (e.g. von Lebieg) for the production function of irrigation (Hexem and Heady 1978, Vaux and Pruitt 1983, Shani and Dudley 2001) do not allow for substitution between water and capital. Moreover, the literature on the productivity of irrigation water is predominantly static in that it considers the empirical relations between aggregate water and yield, but ignores the intra-seasonal distribution of irrigation water.

Biomass growth, however, is a dynamic process (Dinar et al. 1986, Scheierling et al. 1997, Shani et al. 2004, 2005) and the irrigation schedule matters. This suggests that the same amount of aggregate water will produce different yields when distributed differently throughout the growing season. Irrigation technology imposes constraints on irrigation rates and timing. For example, flooding involves intense irrigation events in which large water volumes are applied during short periods, while drip irrigation (connected to a continuous supply source) can be used daily at rates that can be varied (almost) arbitrarily. These differences affect the yields and, as shown in this paper, give rise to substitution between capital and aggregate water, with important policy implications.

In this paper we define and derive a production function of aggregate water and capital, where the latter input affects the degree of control over irrigation scheduling and rates. A larger capital expenditure allows to employ more sophisticated technologies that give rise to higher yields. By definition, a production function specifies the maximal yield that can be produced by each combination of inputs (aggregate water and capital). Therefore, the production function, in the present context, is obtained as the outcome of dynamic optimization problems, in which a given water quota is allocated over time subject to the constraints imposed by irrigation technologies, each requiring a given capital expenditure. We investigate the water-capital substitution of the production function and study implications for input choices, i.e., irrigation water demand and technology adoption. Particularly, we show how water and capital prices affect input choices and the important role of soil type via the drainage process.

2 Irrigation production function

Growers choose from a menu of n feasible irrigation technologies, indexed $i = 1, 2, ..., n$, ranging from flood and furrow through various sprinkler and irrigation machines to sophisticated drip technologies. Technology i requires capital input (expenditure) k_i per unit land (say, hectare). When a fraction α_i of the total land area is irrigated with technology i, the capital expenditure on technology i is $\alpha_i k_i$ per hectare. Let $f_i(q_i)$ represent the maximal yield per hectare attainable with technology i when total water allocation per hectare is q_i. Given per hectare water allocation Q and capital expenditure K, the per hectare irrigation production function is defined as

$$F(K, Q) = \max_{\{\alpha_i, q_i\}} \sum_{i=1}^{n} \alpha_i f_i(q_i) \tag{2.1}$$

subject to $\alpha_i \in [0, 1]$, $\sum_{i=1}^{n} \alpha_i = 1$, $q_i \geq 0$, $\sum_{i=1}^{n} \alpha_i q_i = Q$, $\sum_{i=1}^{n} \alpha_i k_i \leq K$ and (possibly) other feasibility constraints.

A common feasibility constraint entails indivisibility of the irrigation technologies, so that only one technology can be used during a given growing season. This constraint implies that $\alpha_i = 1$ and $q_i = Q$ for some technology while $\alpha_j = 0$ for all $j \neq i$, reducing (2.1) to

$$F(K, Q) = \max_{\{i | k_i \leq K\}} f_i(Q). \tag{2.2}$$

Observe that the definition of the irrigation production function $F(\cdot, \cdot)$ involves a two-step optimization: First, the technology-specific water yield functions $f_i(\cdot)$ are obtained as the result of the optimal temporal distribution of the total water allocation over the irrigation period. Then, the most productive technology (or technology mix) meeting the given capital constraint K is chosen using these functions. The second step must be carried out for each value of Q, because the relative merits of the competing technologies vary with this quantity.

2.1 Water - capital substitution

We seek the water-capital technical rate of substitution associated with the production function (2.2). With a finite number of technologies, the indivisibility constraint implies discrete capital expenditures hence the usual differential treatment does not apply. Let the technology indices $i = 1, 2, ..., n$ be ordered according to the capital input levels associated with each technology such that $k_0 = 0 < k_1 < k_2 < ... < k_n$ ($k_0 = 0$ signifies no irrigation) and define $\triangle k_i \stackrel{\text{def}}{=} k_i - k_{i-1}$. When $K = k_i$, only technologies $0, 1, ..., i$ (with capital expenditure not exceeding k_i) are feasible. Increasing the expenditure by $\triangle K = \triangle k_{i+1}$ adds technology $i + 1$ to the set of feasible technologies.

Suppose that the capital input $K = k_{i-1}$ and the water input Q are employed to produce $F(k_{i-1}, Q)$. Suppose further that the capital input is increased by $\triangle K = \triangle k_i$ to allow the use of technology i. The amount of water ($\triangle Q_i$) that can be saved without compromising the output $F(k_{i-1}, Q)$ is defined by the relation

$$F(k_{i-1}, Q) = F(k_i, Q - \triangle Q_i). \tag{2.3}$$

Typically, the more capital intensive technologies are more productive, hence $\triangle Q_i > 0$. This, however, is not always the case. As we show in Section 4, the effectiveness of each of the various technological constraints manifests different Q-dependence, and for some water allotments the productivity order may be reversed. In such cases increasing capital by $\triangle k_i$ does not contribute to productivity, technology $i - 1$ remains in use and $\triangle Q_i$ vanishes.

The water-capital technical rate of substitution corresponding to technology i for a given water input Q is defined as

$$TRS_i(Q) \stackrel{\text{def}}{=} \triangle Q_i / \triangle k_i. \tag{2.4}$$

2.2 Technology-specific water yield functions: $f_i(Q)$

We adopt the biomass/soil moisture dynamics specified in Shani et al. (2004, 2005). In this framework, $m(t)$ represents the plant biomass at time $t \in [0, T]$, where T denotes the time from emergence to harvest. Marketable yield is given by $y(m(T))$, where $y(\cdot)$ is a non-decreasing yield function . When biomass and yield are the same, $y(m) = m$. Often $y(m)$ vanishes for m

below some threshold biomass level and increases above that threshold. The biomass growth rate at any time $t \in [0, T]$ depends on the current biomass state $m(t)$, the water content in the root zone (soil moisture) $\theta(t)$ as well as on a host of factors including salinity, sunlight intensity, day length and ambient temperature. Taking all factors that are beyond the growers' control as given, biomass growth rate is specified as

$$\dot{m}(t) \stackrel{\text{def}}{=} dm(t)/dt = g(\theta(t))h(m(t)). \tag{2.5}$$

The rate (2.5) is factored into terms depending on θ and m separately. The functions $g(\cdot)$ and $h(\cdot)$ are assumed to be strictly concave, and $g(\cdot)$ obtains a maximum at some finite value $\theta_{\max}$ (too much moisture harms growth). Moreover, $g(\theta_{\min}) = 0$ at the wilting point $\theta_{\min} > 0$. Thus, $g'(\theta_{\max}) = 0$, $g'(\theta) > 0$ for $\theta \in (\theta_{\min}, \theta_{\max})$ and $g''(\theta) < 0$ for all θ.

The water balance in the root zone is determined by water mass conservation, implying that the change in $\theta(t)$ at each point of time is related to the difference between water input through irrigation and losses due to evapotranspiration and drainage:

change in water content = irrigation − evapotranspiration − drainage.

(Rainfall can also be incorporated in this framework, but to focus on irrigation management we assume no rainfall.)

Evapotranspiration rate depends on the states θ and m according to

$$ET(\theta, m) = \beta g(\theta)\varphi(m), \tag{2.6}$$

where the coefficient β represents climatic conditions and $0 \leq \varphi(m) \leq 1$ is a crop scale factor representing the degree of leaves exposure to solar radiation (Hanks 1985). The use of the same factor $g(\theta)$ in (2.5) and (2.6) is based on the linear relation established between biomass production and evapotranspiration (deWit 1958). The drainage rate depends on the soil moisture θ according to the drainage function $D(\cdot)$, assumed increasing and convex.

The irrigation rate using technology i, $x_i(t)$, $t \in [0, T]$, is restricted by three types of feasibility constraints. First, the irrigation rate is constrained to the range $[\underline{x}_i, \overline{x}_i]$ such that $x_i(t)$ either vanishes (no irrigation) or assumes a value within the range. Second, the duration of an irrigation event cannot be shorter than τ_i (an irrigation event is defined as a time interval during which irrigation actually takes place, i.e., $x_i > 0$). The only exception is the final event which can last until all the water allocation has been used up. The third type of feasibility constraints limits the number of irrigation events during one growing season to at most n_i. For example, flood irrigation at a rate of 1200 mm/day with unbounded number of events and minimal event duration of 2 hours is characterized by $\underline{x}_i = \overline{x}_i = 1200$ mm/day, $\tau_i = 2$ hours and $n_i = \infty$. Sprinkle irrigation applied at the rate of 168 mm/day for at least one hour and restricted to no more than five irrigation events is characterized by $\underline{x}_i = \overline{x}_i = 168$ mm/day, $\tau = 1$ hour and $n_i = 5$. A drip technology that can be applied at a freely variable rate up to 48 mm/day is characterized by $\underline{x}_i = 0$ mm/day, $\overline{x}_i = 48$ mm/day, $\tau_i = 0$ and $n_i = \infty$. We let $\Gamma_i = \Gamma(\underline{x}_i, \overline{x}_i, \tau_i, n_i)$ represent the set of all irrigation trajectories feasible under technology i.

When all flow rates are measured in mm/day and θ is a dimensionless water concentration, the soil water balance under technology i is specified as

$$Z\dot{\theta}(t) = x_i(t) - \beta g(\theta(t))\varphi(m(t)) - D(\theta(t)), \tag{2.7}$$

where Z is the depth of the root zone, so that $Z\theta$ measures the total amount of water in the root zone (mm).

Technology i's water yield function, $f_i(Q)$, is the maximal harvested yield attainable with this technology when aggregate water allocation is Q:

$$f_i(Q) = \max_{\{x_i(t),\, t\in[0,T]\}} \{y(m(T))\} \tag{2.8}$$

subject to (2.5), (2.7), $x_i(t) \in \Gamma_i$ and

$$\int_0^T x_i(t)dt \le Q, \tag{2.9}$$

given the initial biomass and soil moisture $m(0) = m_0$ and $\theta(0) = \theta_0$. (Focusing interest on the output effects of capital and water inputs, we suppress m_0 and θ_0 as arguments of f_i.) The technological properties of drip irrigation allow an analytic derivation of the optimal irrigation policy $x^*(t)$ (see Shani et al. 2004, 2005), from which $f(\cdot)$ is obtained for this technology. The solution of (2.8) for the other irrigation techniques requires finding the time and duration of each irrigation event, using numerical optimization methods. In Section 4 we solve for the optimal irrigation policy and the resulting water-yield functions of four common irrigation technologies.

3 Water demand and technology choice

With output price normalized to unity and c and r representing water price and capital rental rate, respectively, the profit generated by K and Q is

$$F(K,Q) - cQ - rK.$$

Profit-seeking, price-taking growers choose the water and capital (irrigation technology) inputs that maximize profit. If only one technology can be used during a certain growing period, this task can be divided into two stages: first, find the water demand for each technology; then, choose the optimal irrigation technology. We discuss each stage in turn.

3.1 Technology-specific water demand

We seek the derived demand for irrigation water by growers using technology i. Let $\mu_i(Q)$ represent the shadow price of the water constraint (2.9), i.e., $\mu_i(Q)$ measures the output increment associated with a small (marginal) increase in Q:[1]

$$\mu_i(Q) = f_i'(Q). \tag{3.1}$$

(It is assumed that $f_i(Q)$ is differentiable above the threshold allotment $\underline{Q}_i$ below which yield vanishes). Thus, $\mu_i(Q)$ is the inverse derived demand for irrigation water under technology i. To see this, note that when the price of water (relative to output price) is $c = \mu_i(Q)$, price-taking growers using technology i demand the quantity Q that maximizes $\{f_i(Q) - cQ\}$ by satisfying $f_i'(Q) = c$, and the claim follows from (3.1).

Typically $f_i'(Q)$ is decreasing for Q above $\underline{Q}_i$ (due to diminishing marginal productivity of water – see Figures 1 and 4) and the derived demand for irrigation water with technology i is given by

$$Q_i(c) = \begin{cases} \mu_i^{-1}(c) & \text{if } c \in [0, \mu_i(\underline{Q}_i)] \\ 0 & \text{if } c > \mu_i(\underline{Q}_i). \end{cases} \tag{3.2}$$

[1]Introducing $Q(t) = Q - \int_0^t x(s)ds$ as an additional state variable measuring the remaining water quota available at time t (with $\dot{Q}(t) = -x_i(t)$, $Q(0) = Q$ and $Q(T) \geq 0$), and recalling the interpretation of the costate variable $\mu(\cdot)$ as the derivative of the value function (when the latter is differentiable) gives (3.1).

3.2 Technology choice

In view of (3.2), the (per hectare) profit function for technology i is

$$\pi_i(c, r) = f_i(Q_i(c)) - cQ_i(c) - rk_i. \tag{3.3}$$

The chosen irrigation technology i^* is the one that yields the highest profit:

$$\pi_{i^*}(c, r) = \max_{\{i=1,2,\ldots,n\}} \{\pi_i(c, r)\}. \tag{3.4}$$

(Ties are broken by some prespecified rule.) Observe that the technology choice (3.4) implies renting the capital stock $K = k_{i^*}$. Indeed, had the technology choice been carried out over an infinite menu, with k serving as a continuous capital index associated with each technology, the selection rule (3.4) could be interpreted as determining the irrigation capital K by equating the rental rate r with the shadow price associated with the constraint $k \leq K$.

We can now see how water price (representing extraction and conveyance costs as well as a scarcity rent) and the price of capital affect technology adoption decisions and water demand. A higher water price reduces water input (see 3.2) and with smaller allocations the output advantages of water-efficient technologies, such as drip, over water-lavish technologies, such as flood, are more pronounced (i.e., differences between the water yield functions are larger – see Section 4). Thus, higher water prices encourage adoption of water-efficient technologies. Such technologies, however, are often more capital intensive and the output gain should be sufficient to compensate the added capital cost for adoption to pay off. Increasing the capital rental rate r renders such compensation less likely to occur, hence discourages adoption of water-saving technologies. In the following section we investigate these issues via a real world example.

4 Application

The crop considered is Ornamental Sunflower (*Helianthus annuus* var dwarf yellow) grown in the Arava Valley, Israel. Lack of precipitation throughout the growing period and deep groundwater (120 m below soil surface) imply that irrigation is the only source of water.

4.1 Irrigation technologies

Four irrigation technologies are feasible in this study: flood ($i = 1$), sprinkle restricted to five irrigation events ($i = 2$), sprinkle restricted to ten irrigation events ($i = 3$) and drip ($i = 4$). The technological constraints are summarized in Table 1.

Table 1: Irrigation technology parameters.

i	Technology	$\underline{x}_i$ (mm/day)	$\overline{x}_i$ (mm/day)	τ_i (hour)	n_i
1	Flood	1200	1200	2	∞
2	Sprinkle 5	168	168	2	5
3	Sprinkle 10	168	168	2	10
4	Drip	0	48	0	∞

4.2 Biomass dynamics

Following Shani et al. (2004), the $g(\cdot)$ and $h(\cdot)$ functions are specified as

$$g(\theta) = 1.21\Theta - 1.71\Theta^2, \tag{4.1}$$

with

$$\Theta \stackrel{\text{def}}{=} (\theta - 0.09)/0.31 \tag{4.2}$$

corresponding to the wilting point $\theta_{\min} = 0.09$ (where the growth rate vanishes), and

$$h(m) = m(1 - m/491). \tag{4.3}$$

The biomass state equation (2.5) becomes

$$\dot{m} = (1.21\Theta - 1.71\Theta^2)m(1 - m/491) \tag{4.4}$$

4.3 Soil moisture dynamics

The drainage function is of the form (see Brooks and Corey 1964)

$$D(\theta) = K_S\Theta_D^{\eta}, \tag{4.5}$$

where K_S is the hydraulic conductivity,

$$\Theta_D \stackrel{\text{def}}{=} (\theta - \theta_R)/(\theta_S - \theta_R), \tag{4.6}$$

θ_S and θ_R are the saturated and residual water content, respectively, and $\eta > 1$ is the drainage exponent. The four parameters (K_S, θ_R, θ_S and η) vary with the soil type. We consider two soil types: sandy loam (which is the one actually prevailing in the Arava Valley) and loam. We use the estimates of Shani et al. (1987) as the empirical parameter values for these soils; these values are summarized in Table 2.

Table 2: Drainage parameters for loam and sandy loam soils.

Parameter	loam	sandy loam
K_S	1200	3600
θ_S	0.45	0.4
θ_R	0.04	0.04
η	8	5.73

With $\beta = 37.3$ mm, $\varphi(m) = m(1-m/785.6)/196.4$ and $Z = 600$ mm taken from Shani et al. (2004), the soil moisture dynamic equation (2.7) assumes the form

$$\dot{\theta} = [x_i - 0.19(1.21\Theta - 1.71\Theta^2)m(1 - m/785.6) - D(\theta)]/600 \tag{4.7}$$

where Θ is defined in (4.2) and $D(\theta)$, defined in (4.5), varies between the soil types according to the parameters of Table 2.

4.4 Yield - biomass specification

Marketable yield for sunflowers is obtained only at biomass levels above $350\,g/m^2$. At the maximal biomass ($m = 491\,g/m^2$) the yield comprises 80% of the biomass. Assuming a linear increase gives rise to the following yield function

$$y(m) = \begin{cases} 0 & \text{if } m < 350\ g/m^2 \\ 2.79(m - 350) & \text{if } m \geq 350\ g/m^2. \end{cases} \tag{4.8}$$

4.5 Simulation results

The initial soil water and biomass levels are taken at $\theta_0 = 0.1$ (just above the wilting point) and $m_0 = 10\ g/m^2$ (about 2% of the maximal obtainable biomass). The yield is harvested after a growing period of $T = 45$ days.

4.5.1 Sandy loam soil

Figure 1 displays the water yield functions $f_i(\cdot),\ i = 1, 2, 3, 4$, obtained with sandy loam. For drip irrigation, the minimal water allotment required to obtain a positive yield ($\underline{Q}_4$) is about 120 mm. Since drip irrigation is the most efficient technology, this minimal quantity is the threshold below which the production function vanishes for any value of K. With total water allocation exceeding 120 mm, the optimal drip policy is to reach a certain moisture value $\hat{\theta}$ as rapidly as possible by irrigating at the maximal feasible rate. Once $\hat{\theta}$ has been reached, irrigation rate is tuned so as to maintain moisture fixed at this state. Finally, at some time $t < T$, irrigation is ceased until the harvest date. Increasing the allotment Q allows to raise the fixed moisture state $\hat{\theta}$ and to reduce the duration of the final dry period. The diminishing marginal productivity of water is evident in the Figure: raising Q from 200 mm to 300 mm increases the drip yield by 93 g/m^2, whereas the same raise from $Q = 600$ mm generates a yield increase of less than 3 g/m^2.

The water - capital rates of substitution, $TRS_i(Q)$, can be read off the water yield curves. For example, the output obtained with $Q = 221$ mm using Sprinkle 10 ($i = 3$) is the same as the output obtained with $Q = 300$ mm using Sprinkle 5 ($i = 2$) hence $\triangle Q_3 = 79$ mm (Figure 1). Thus, noting (2.4), $TRS_3(300) = 79/\triangle k_3$, where $\triangle k_3$ is the difference between the capital expenditures on the two sprinkle technologies.

The crossing of the curves corresponding to the Flood and Sprinkle 5 technologies illustrates the way in which the technological constraints interfere with the growth process. Both technologies limit the number of irrigation events, albeit via different mechanisms. The Sprinkle 5 technology allows only five irrigation events as an intrinsic technological constraint. The high irrigation rate under Flood, combined with the minimal event duration, imply that a significant fraction of the total water allotment is used in a single irrigation

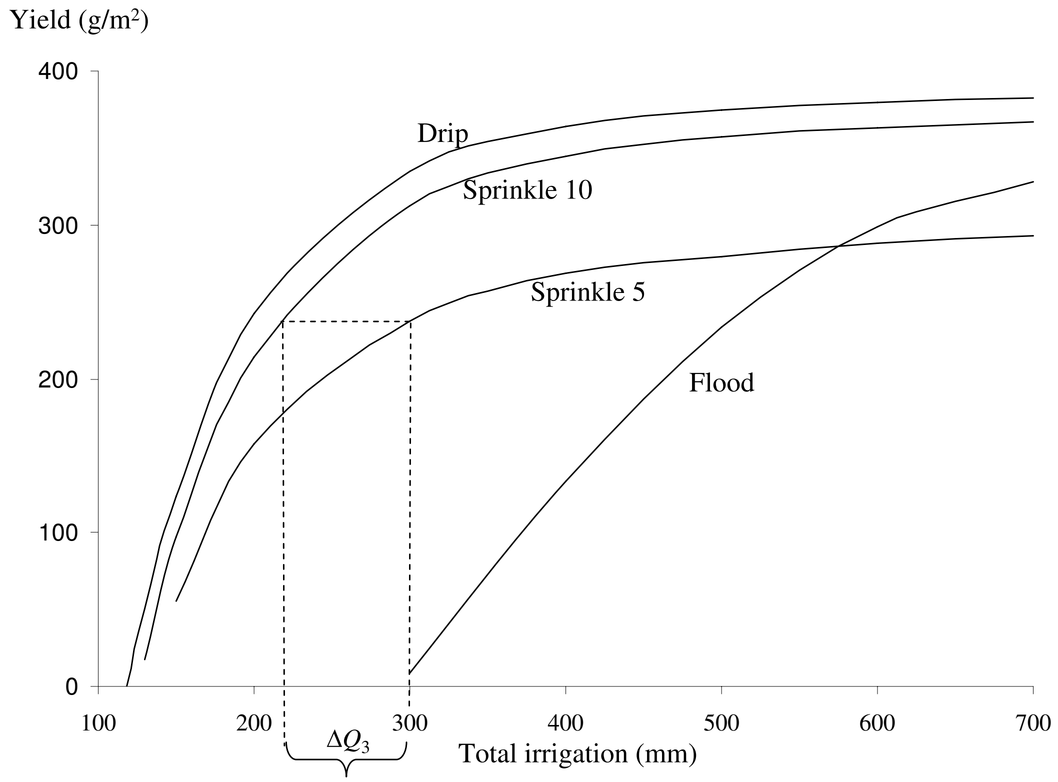

Figure 1: Sandy loam water yield functions $f_i(\cdot)$ vs. water allotment Q. $F(k_2, 300) = f_2(300) = f_3(221) = F(k_3, 221)$ hence $\triangle Q_3 = 300 - 221 = 79$mm and $TRS_3(300) = 79/\triangle k_3$, where $\triangle k_3 = k_3 - k_2$.

event. Thus, a small value of Q permits only a few (flood) irrigation events. For both technologies, a small number of irrigation events corresponds to a large variation in water contents about the desired level of θ, with significant drainage losses when θ is well above the average level and low growth rates when θ falls below it during the long time intervals extending between the events. (Indeed, this is the source of advantage of Sprinkle 10 over Sprinkle 5.) Below $Q = 500$ mm, the Flood technology allows the smallest number of events which corresponds to the minimal yield. Above this allotment, the technological constraint of Sprinkle 5 implies that this technology has the smallest number of irrigation events, rendering it the least productive technology and explaining the crossing of the Sprinkle 5 and Flood yield curves.

Figure 2 shows the soil moisture trajectories of the four irrigation technolo-

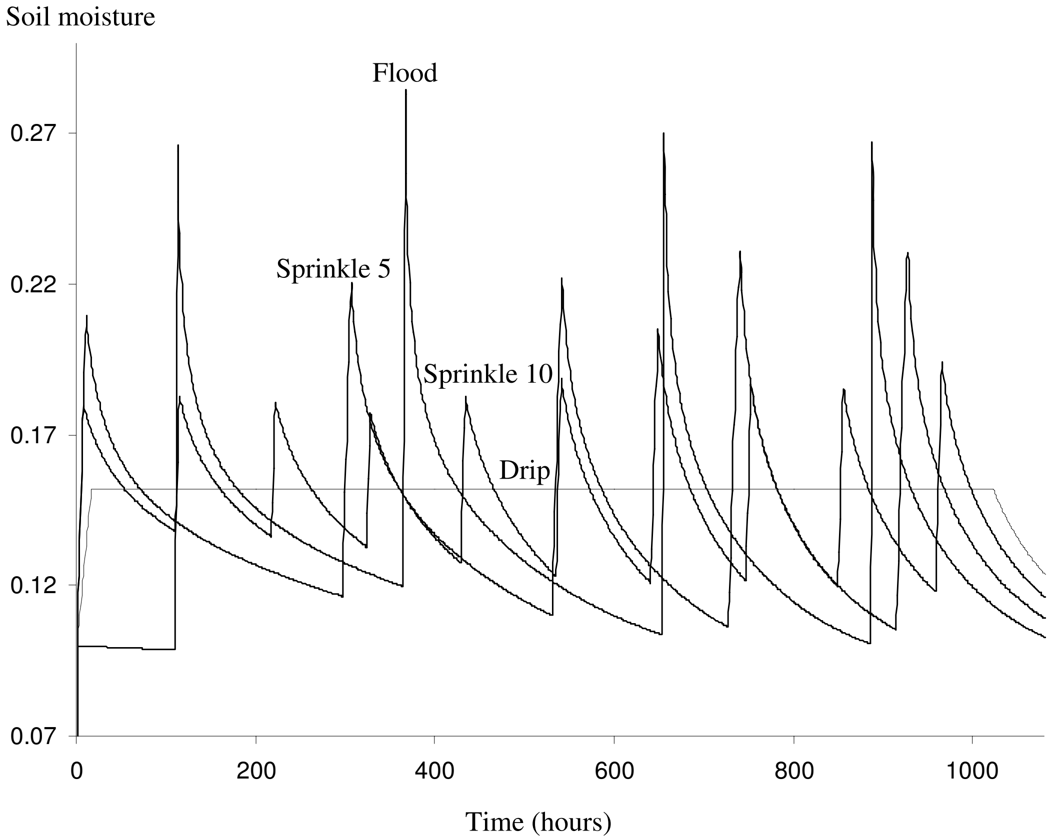

Figure 2: Optimal moisture profiles in sandy loam with $Q = 400$ mm.

gies at $Q = 400$ mm. The trajectories associated with sprinkles and flooding display large oscillations, in contrast to the constant θ policy that characterizes drip irrigation during most of the growing period. The oscillation amplitude is strongly correlated with the number of irrigation events, with Flood (that allows only four events at this value of Q) showing the largest amplitude. This explains the relative ranking in productivity reported in Figure 1 for this water allotment. In fact, this ranking can be traced to the strong non-linearity of the drainage term $D(\cdot)$ that accounts for water lost mostly during the high-moisture period. With few events and large amplitude θ-oscillations, drainage consumes a significant fraction of the total allotment (see Figure 3) leaving less water to meet the needs of the growing plants and reducing the productivity of irrigation water.

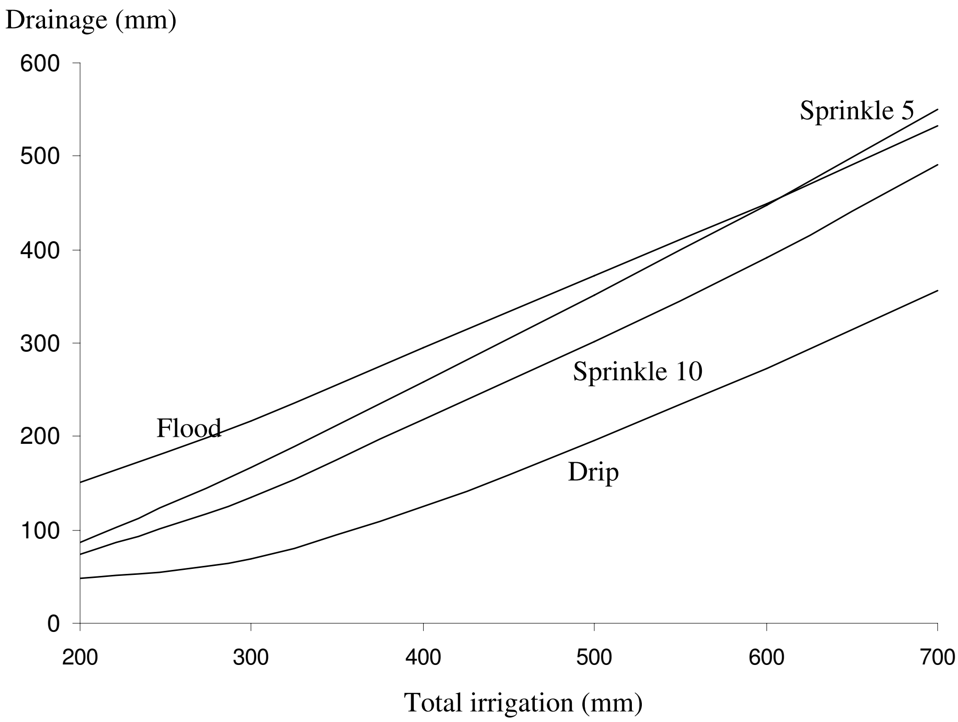

Figure 3: Drainage losses in sandy loam vs. water allotment Q.

The clear correspondence between the results displayed in Figures 1 and 3 confirms that the differences in water productivity derived for various technologies are mostly due to drainage losses. This observation suggests that heavier soils, where drainage rates are significantly lower, should leave less room for technological productivity enhancement.

4.5.2 Loam soil

Figure 4 verifies our expectation by showing the water-yield functions $f_i(\cdot)$ obtained for the heavier loam soil. Comparing with Figure 1, we see that water is more productive for all technologies ($Q = 350$ mm suffices to produce the maximal yield with all technologies) and that the differences between the yield curves are much smaller than those obtained for sandy loam. In fact, these differences between drip and sprinkling are below the numerical accuracy of the simulations. For heavier soils, then, the adaptation of the more advanced technologies can be justified only with low capital costs or under severe water scarcity.

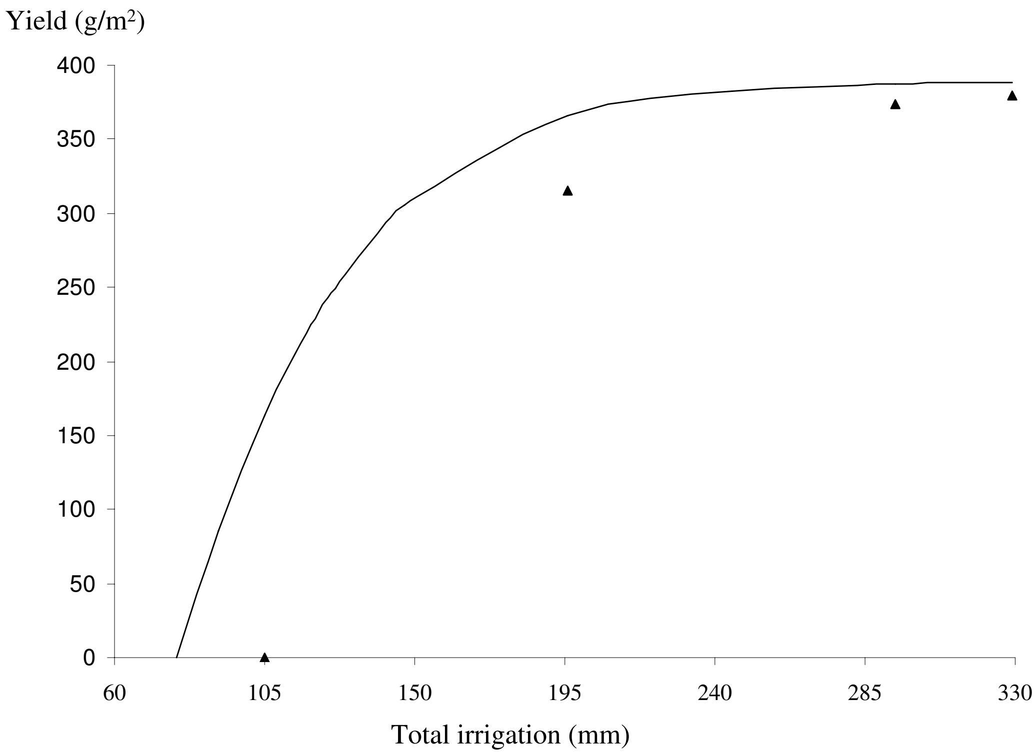

Figure 4: Loam water yield functions for Drip (solid line) and Flood (triangle symbols) vs. water allotment Q.

The results underscore the distinction between the time at which water is applied and the time it is actually consumed for evapotranspiration. When drainage is significant, a large time gap implies water loss and reduced yield. Irrigation rates, then, should be well adjusted to the varying instantaneous needs of the growing plants. This goal can be achieved only with the highly flexible drip technology. When the drainage term is small (as in the loam soil considered here) the soil serves as a water reservoir, keeping the moisture from the time of the intense irrigation events until it is taken by the roots. The timing constraints of the simpler technologies bear small losses, and large capital investments are not worthwhile.

5 Concluding comments

Sophisticated irrigation technologies allow to adjust irrigation scheduling to the varying needs of the plants along the growing period, enhancing the productivity of irrigation water. This simple observation gives rise to irrigation production functions that exhibit water-capital substitution, with important implications regarding irrigation water demand and irrigation technology adoption. The application of advanced technologies requires capital investments that can be justified only when the enhanced yields compensate for the extra cost of capital. The latter is more likely to occur when water is expensive, when capital rental rate is small and for soils with high drainage rates that claim a significant fraction of the applied water. These considerations are investigated analytically and demonstrated for a particular crop.

The drainage factor becomes more influential when environmental considerations are incorporated. Drainage water carries along dissolved fertilizers and pesticide materials which contaminate the soil and underlying groundwater. Accounting for such damages entails pricing drainage water over and above the price of irrigation water, increasing the profitability of water-saving technologies, compared with water-lavish technologies.

Possible extensions include treating water of different quality as an additional input as well as allowing for other inputs such as fertilizers or pesticides. Often, the use of some amounts of locally-available brackish water can substitute for scarce high quality water at little cost in terms of yield (Shani et al. 2005). This possibility will reduce the attractiveness of adopting the more expensive technologies by relaxing constraints on available freshwater allotments. The corresponding production function should include both fresh and brackish water as independent variables, and the biomass growth equation should include the dependence on the salinity of the water mix used for irrigation. These extensions reinforce the main message of this work: optimal irrigation policies must take into account the dynamic nature of the processes of biomass growth and moisture evolution in unsaturated soils and these processes depend critically on irrigation scheduling, which in turn is restricted by the applied irrigation technology. This observation gives rise to the water-capital substitution in irrigation production processes with far reaching implications regarding irrigation water use and technology adoption.

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