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REGULATING ENVIRONMENTAL THREATS

by

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Abstract

Environmental consequences of natural resource exploitation often entail threats of future occurrences of detrimental abrupt events rather than (or in addition to) inflicting a damage gradually. The possibility of abrupt occurrence of climate-change related calamities is a case in mind. The uncertainty associated with the realization of these threats and their public-bad nature complicate the determination of optimal economic response. We analyze the regulation of such environmental threats by means of a Pigouvian hazard tax, based on the shadow cost of the hazard-generating activities. A numerical example illustrates possible effects of the proposed regulation scheme.

Keywords: environmental events; emission; climate change; regulation; Pigouvian tax; hazard rate; uncertainty

JEL Classification: H23; H41; O13; Q54; Q58

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1 Introduction

Environmental Pigouvian taxes are based on marginal external damages. Yet, the derivation of such taxes is far from obvious when resource exploitation, rather than (or in addition to) implying gradual degradation, poses environmental threats concerning abrupt events triggered at an unpredictable future date by conditions that are not fully understood. Sudden occurrences of this nature are related to nonlinear phenomena such as positive feedbacks, hysteresis and the presence of thresholds that are ubiquitous in environmental processes (Mäler 2000, Dasgupta and Mäler 2003, Brock and Starrett 2003). A case in mind is climate change. Gradual atmospheric accumulation of greenhouse gases (GHG) is thought to cause or accelerate global warming processes which may trigger catastrophic events (Broecker 1997, IPCC 2001, Alley et al. 2003). The uncertainty regarding the occurrence date and extent of damage inflicted by climate-change driven events as well as the common pool nature of the atmosphere hamper effective environmental regulation, as the ongoing debate regarding the Kyoto Protocol attests (Nordhaus and Boyer 2000, Nordhaus 2001).

In this work we study the regulation of environmental threats associated with abrupt events. For the sake of concreteness we focus on the example of climate change, but note that the analysis extends to other exploitation-induced hazards, as those discussed in Tsur and Zemel (2004, 2006). We consider an economy whose production activities use an intermediate input (energy) that can be derived from polluting (fossil) sources, which exacerbate environmental hazards, or from clean (solar) sources. The use of the former entails emissions that accumulate to form a stock (atmospheric GHG concentration).

No immediate harm is inflicted by the emissions per se nor by the resulting stock accumulation. However, the accumulated stock affects the probability of occurrence of events of detrimental consequences. We refer, therefore, to the corresponding source of the intermediate input as 'hazardous'. What is the competitive temporal allocation of resources in the economy? How should the economy allocate resources in light of the hovering environmental threats? What kind of regulation can be used to implement the latter? These are the questions that concern us here.

To address these issues we develop a framework that incorporates occurrence threats of discrete events (as in Tsur and Zemel 1996, 1998, 2004, and Nævdal 2006) within a multi-sector economy that derives inputs from hazardous and from clean sources . By contrasting the competitive against the socially optimal regime we identify the shadow cost associated with the public bad feature of the environmental threat and define the ensuing Pigouvian hazard tax. The latter, then, is used to specify a regulation scheme that implements the socially optimal allocation.

Our work is related to the broad literature on environmental regulation and taxation (see, e.g., Bovenberg and Goulder 1996, Goulder et al. 1999), but derives mainly from the (growing) body of research on the economics of climate change, advanced by Nordhaus and collaborators (see Nordhaus and Boyer 2000, and references therein). This research offers a variety of models, disaggregated by geographical regions, economic sectors and technologies (see also Chakravorty et al. 1997). Virtually all of this literature, however, assumes gradual environmental degradation and ignores the possibility of abrupt events. To allow a sharp focus on abrupt events, we simplify by considering an aggregate framework (as in Tsur and Zemel 2005).

The next section describes the economy under environmental threat, formulated in terms of a stock-dependent hazard rate that measures the probability of sudden occurrence of the detrimental event. Section 3 characterizes the competitive allocation obtained when agents treat the environmental hazard as a public bad and ignore their own contributions to its accumulation. Section 4 offers a Pigouvian hazard tax for a regulation scheme that implements the socially optimal allocation. In section 5 we analyze steady state behavior and provide a numerical illustration of possible effects of the externalities associated with the environmental threats. Section 6 concludes.

2 The economy

The economy consists of households that own capital and labor, a final (consumption) good manufacturing sector and two intermediate good sectors. The final good is manufactured by means of capital (K), labor (L) and an intermediate input (X, e.g., energy) that can be derived from any of two substitute sources, which differ in the way they affect the surrounding environment. One intermediate input (X_1, e.g., fossil energy) is hazardous in that its use involves emissions that accumulate to enhance the hazard of abrupt occurrence of some detrimental event. The second intermediate input (X_2, e.g., solar energy) is clean and entails no adverse environmental byproducts.

The use of the hazardous input at the rate X_1 entails emission at the rate $e(X_1)$ of pollutants which accumulate in the form of the hazardous stock Q according to

$$\dot{Q}(t) = e(X_1(t)) - \delta Q(t) \tag{2.1}$$

where δ is a natural decay (or environmental cleansing) parameter. The environmental effect of Q is manifest via the hazard rate $h(Q)$, such that $h(Q)dt$ measures the conditional probability that the event will occur during the time interval $[t, t+dt]$ given that it has not occurred by time t when the stock is Q.

Let T represent the random event-occurrence time, with the probability distribution and density functions $F(t)$ and $f(t)$, respectively. For a given $Q(t)$ process, the hazard rate $h(Q(t))$ is related to the distribution $F(t)$ according to $h(Q(t)) = f(t)/(1-F(t)) = -d[\ln(1-F(t))]/dt$, yielding

$$F(t) = 1 - e^{-\Omega(t)} \text{ and } f(t) = h(Q(t))e^{-\Omega(t)}, \tag{2.2}$$

where

$$\Omega(t) = \int_0^t h(Q(\tau))d\tau \tag{2.3}$$

is the accumulated hazard. Upon occurrence at time T, the event inflicts some damage, as explained below.

3 Competitive allocation

We characterize here the allocation process when firms and households operate in a competitive environment.

3.1 The final good sector

The final good sector consists of many firms, each seeking to maximize instantaneous profit at any point of time. Firm i uses capital (K_i), labor (L_i) and an intermediate input ($X_i = X_{1i} + X_{2i}$) to produce output (Y_i) according to the linearly homogenous production function $G(K_i, X_i, L_i)$, where X_{1i} and X_{2i} denote rates of use of the hazardous and clean intermediate inputs, respectively.[1] In the competitive environment, firms take as given the capital rental rate (r), the prices of the intermediate inputs (p_1 and p_2), and the wage rate (w) and plan production in order to maximize the instantaneous profit

$$G(K_i, X_i, L_i) - rK_i - p_1 X_{1i} - p_2 X_{2i} - wL_i = L_i[g(k, x) - rk - p_1 x_1 - p_2 x_2 - w]$$

where lowercase letters represent per worker (or per capita) variables, e.g., $k = K_i/L_i$. The per worker variables are the same across firms that use the same technology (hence the firm subscript i is dropped) and $g(k, x) = G(k, x, 1)$. Summing over all firms gives the aggregate profit

$$\Pi = L[g(k, x) - rk - p_1 x_1 - p_2 x_2 - w], \tag{3.1}$$

where $L = \sum_i L_i$ is the population size (or the number of workers).

Denoting the marginal productivities of capital and intermediate input by $g_k \equiv \partial g/\partial k$ and $g_x \equiv \partial g/\partial x$, respectively, the necessary conditions for profit maximization include

$$g_k(k, x) = r \tag{3.2}$$

and

$$g_x(k, x) = p \equiv \min(p_1, p_2). \tag{3.3}$$

$g_x(k, X/L)$ is the (inverse) derived demand for the intermediate input X and is decreasing in X (since $g_{xx} < 0$) and assumed to increase in k ($g_{xk} > 0$).

Since the intermediate inputs are perfect substitutes, firms will use only the cheaper input if $p_1 \neq p_2$ and will use both (or be indifferent between using either) when $p_1 = p_2 = p$. Thus, $p_1 x_1 + p_2 x_2 = px$ in all cases. The wage rate that clears the labor market gives rise to a vanishing profit, implying, in

[1] In addition to linear homogeneity, the production function satisfies the usual properties of concavity and positive-diminishing marginal productivity.

view of (3.1),

$$g(k,x) - rk - px = w. \tag{3.4}$$

3.2 The intermediate good sectors

The marginal cost of manufacturing the clean intermediate good X_2 is assumed constant at p_2; the supply curve of X_2 is thus the horizontal line at the level p_2. The hazardous input is manufactured with an increasing and strictly convex cost function $Z(X_1)$ and its supply curve is the upward sloping marginal cost curve $Z'(X_1)$.[2] We assume that $Z(0) = 0$ and $Z'(0) < p_2$. The supply curve of the intermediate input is given by

$$m(X) = \min(Z'(X), p_2). \tag{3.5}$$

The intermediate input market is in equilibrium when supply equals demand, i.e., when

$$m(X) = g_x(k, X/L). \tag{3.6}$$

Define $X(k)$ to be the rate X that satisfies (3.6) for a given capital stock k and let X_1^c be the rate at which $m(X)$ switches from $Z'(X)$ to p_2 (see Figure 1):

$$X_1^c = Z'^{-1}(p_2). \tag{3.7}$$

The properties of Z imply that $X_1^c > 0$. Given k, the intermediate input is used at the rate $X(k)$ and is allocated between the hazardous and clean inputs according to (Figure 1)

$$(i)\ X_1(k) = \min(X(k), X_1^c) \ \text{ and } \ (ii)\ X_2(k) = X(k) - X_1(k). \tag{3.8}$$

[2]The assumptions regarding the marginal costs stem from the observation that fossil energy technology is mature and holds a dominant market share. Further increase in supply must resort to less efficient and more expensive sources. Renewable energy technologies, in contrast, are not yet fully developed and increased use may actually reduce prices. For a detailed description see Chakravorty et al. (1997).

Notice that if $X(k) \leq X_1^c$, only X_1 is used at a price $Z'(X(k))$, whereas when $X(k) > X_1^c$ both inputs are supplied at a price p_2.

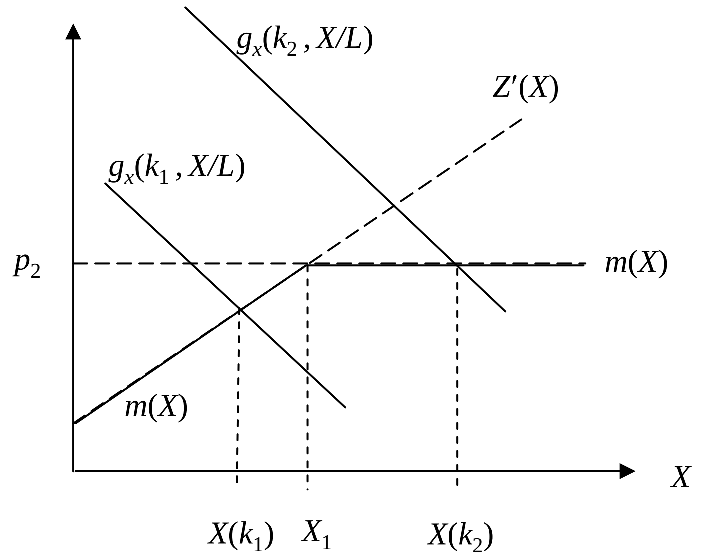

Figure 1: Given the capital stock k_1, the market allocation of intermediate input is $X(k_1)$, supplied solely from the hazardous source. With a larger capital $k_2 > k_1$, the allocation is $X(k_2)$, of which X_1 is supplied from the hazardous source and $X(k_2) - X_1$ from the clean source.

The instantaneous profit of the hazardous input sector is

$$\int_0^{X_1(k)} [m(X(k)) - Z'(X)]dX = m(X(k))X_1(k) - Z(X_1(k)) = pX_1(k) - Z(X_1(k)),$$

where $m(X(k)) = p$ follows from (3.3) and (3.6). This profit is readily interpreted as the difference between the revenues derived from supplying $X_1(k)$ at the price p and the cost $Z(X_1(k))$ of manufacturing it. The per capita rent generated by the intermediate sector is thus given by

$$[pX_1(k) - Z(X_1(k))]/L. \tag{3.9}$$

No profit is forthcoming from the clean input sector.

3.3 Households

Households derive income from interest on savings, labor wage and intermediate input rent. At any given time t, an household's income equals $rk(t)+w(t)+[pX_1(k(t))-Z(X_1(k(t)))]/L$ and is allocated between consumption $c(t)$ and net saving $\dot{k}(t)$. Using (3.4), the household budget constraint becomes

$$\dot{k}(t) = g(k(t), x_1(t)+x_2(t)) - p_2 x_2(t) - z(x_1(t)) - c(t), \tag{3.10}$$

where $z(x_1(t)) = Z(Lx_1(t))/L$.

The consumption rate $c(t)$ generates the instantaneous utility $u(c(t))$ for some increasing and strictly concave utility function $u(c)$. A feasible consumption-saving process $\{c(t), k(t)\}$ gives rise to the aggregate intermediate input processes $X_1(t) = X_1(k(t))$ and $X_2(t) = X(k(t)) - X_1(t)$ (cf. equation (3.8)), which in turn generates emission at the rate $e(X_1(t))$ that drives the hazardous stock process $Q(t)$ via (2.1).

The present value generated by a feasible $\{c(t), k(t)\}$ plan, interrupted at time T upon occurrence, is

$$\int_0^T u(c(t))e^{-\rho t}dt + e^{-\rho T}\varphi(k(T), Q(T)),$$

where ρ is the utility discount rate and $\varphi(k(T), Q(T))$ is the post-event value, representing the maximal present value from the occurrence time T onward discounted to time T (several possible specifications for the post-event value are described below). Denoting the expectation with respect to the distribution of the random occurrence time T by E_T, the expected present value is evaluated as

$$E_T\left\{\int_0^T u(c(t))e^{-\rho t}dt + e^{-\rho T}\varphi(k(T),Q(T))\right\} =$$
$$\int_0^\infty \{u(c(t))[1-F(t)] + f(t)\varphi(k(t),Q(t))\}\, e^{-\rho t}dt =$$
$$\int_0^\infty [u(c(t)) + h(Q(t))\varphi(k(t),Q(t))]e^{-\Gamma(t)}dt, \qquad (3.11)$$

where

$$\Gamma(t) = \rho t + \Omega(t) = \int_0^t [\rho + h(Q(s))]ds \qquad (3.12)$$

incorporates the hazard rate into the effective discount rate $\rho + h(Q)$.

The household's consumption-saving plan is determined according to

$$v(k,Q) = \max_{\{c(t)\}} \int_0^\infty [u(c(t)) + h(Q(t))\varphi(k(t),Q(t))]e^{-\Gamma(t)}dt \qquad (3.13)$$

subject to (3.10), given the initial stocks $k(0) = k$, $Q(0) = Q$ and various feasibility (e.g., nonnegativity) constraints. In solving (3.13), individual households take the aggregate quantities $X_1(k(t))$, $X_2(k(t))$, $Q(t)$ and $\Gamma(t)$ parametrically: the rates $X_1(k)$ and $X_2(k)$ are determined by the intermediate-input market conditions (3.6) and (3.8), while $Q(t)$ evolves according to (2.1) and $\Gamma(t)$ is defined in (3.12).[3]

The event threat enters the household problem via the effective discount rate $\rho + h(Q)$ and through the expected post-event value $h(Q)\varphi(k,Q)$ on the right-hand side of (3.13). The post-event value $\varphi(k,Q)$ is related to a damage function defined as

$$\psi(k,Q) = v(k,Q) - \varphi(k,Q), \qquad (3.14)$$

[3]This is essentially the competitive behavior assumption, which without external effects and other market imperfections gives rise to efficient allocation. However, the hazardous stock Q acts here as a public bad, rendering the competitive allocation suboptimal. The socially optimal allocation is characterized in the following section.

such that the expected loss associated with occurrence during the time interval $[t, t + dt]$ is $\psi(k(t), Q(t))h(Q(t))dt$. For example, a "doomsday" event that ceases all further economic activities entails $\varphi(k, Q) = 0$ and $\psi(k, Q) = v(k, Q)$. Recurrent events that destroy some amount D_k of the existing capital stock give rise to $\varphi(k, Q) = v(k - D_k, Q)$. When the extent of damage is also subject to uncertainty, we take ψ to represent its expected value.

The Hamiltonian corresponding to (3.13) is (the time argument is suppressed for brevity)

$$H = [u(c) + h(Q)\varphi(k, Q)]e^{-\Gamma} + \lambda[g(k, x_1 + x_2) - p_2 x_2 - z(x_1) - c].$$

The necessary conditions for optimum include

$$u'(c)e^{-\Gamma} - \lambda = 0 \tag{3.15}$$

and

$$\dot{\lambda} = -h(Q)\varphi_k(k, Q)e^{-\Gamma} - \lambda g_k(k, x). \tag{3.16}$$

These conditions, together with the transversality condition that $\lambda(t)k(t)$ vanishes asymptotically, determine the competitive policy $\{\lambda(t), c(t), k(t)\}_{t=0}^{\infty}$, assumed unique.[4]

4 Regulation

The public bad nature of the hazardous stock Q implies that the competitive allocation is suboptimal. In this section we offer a regulation scheme that implements the socially optimal allocation. To that end, we identify the external hazard cost of emission and use it to specify a Pigouvian hazard tax. We begin by characterizing the optimal allocation.

[4]For conditions ensuring a unique solution in infinite horizon dynamic optimization see, e.g., Caputo (2005, Chapter 14).

4.1 Optimal allocation

The socially optimal policy internalizes the external hazard effects by choosing the rates of intermediate inputs according to

$$V^s(K,Q) = \max_{\{c(t),x_1(t),x_2(t)\}} \int_0^\infty L[u(c(t)) + h(Q(t))\varphi(k(t),Q(t))]e^{-\Gamma(t)}dt \quad (4.1)$$

subject to (2.1), (2.3) and the aggregate version of (3.10)[5]

$$\dot{K}(t) = L[g(k(t), x_1(t) + x_2(t)) - p_2 x_2(t) - z(x_1(t)) - c(t)], \qquad (4.2)$$

given the usual initial and feasibility conditions.

Suppressing again the time argument from all functions, the Hamiltonian associated with (4.1) is

$$\begin{aligned} H^s = L[u(c) + h(Q)\varphi(k,Q)]e^{-\Gamma} \quad &+ \quad \lambda^s L[g(k, x_1 + x_2) - p_2 x_2 - z(x_1) - c] \\ &+ \quad \gamma[e(Lx_1) - \delta Q] + \mu h(Q), \end{aligned}$$

where λ^s, γ and μ are the costate variables associated with $K(= Lk)$, Q and the hazard stock Ω, respectively. The necessary conditions for an interior optimum (with $x_2 > 0$), expressed in terms of per capita quantities for ease of comparison with the competitive allocation, include

$$u'(c)e^{-\Gamma} = \lambda^s \qquad (4.3)$$

$$g_x(k, x_1 + x_2) = z'(x_1) - \frac{\gamma}{\lambda^s} e'(Lx_1), \qquad (4.4)$$

$$g_x(k, x_1 + x_2) = p_2, \qquad (4.5)$$

$$\dot{\lambda}^s = -\frac{\partial H^s}{\partial K} = -\frac{1}{L}\frac{\partial H^s}{\partial k} = -h(Q)\varphi_k(k,Q)e^{-\Gamma} - \lambda^s g_k(k, x_1 + x_2), \qquad (4.6)$$

$$\dot{\gamma} = -L\left\{h'(Q)\varphi(k,Q) + h(Q)\varphi_Q(k,Q)\right\}e^{-\Gamma} + \gamma\delta - \mu h'(Q) \qquad (4.7)$$

[5] We assume that labor L is constant. Exogenous population growth and labor augmenting technical change can be added with minor modifications.

and (recalling (3.12))

$$\dot{\mu} = L[u(c) + h(Q)\varphi(k, Q)]e^{-\Gamma}. \tag{4.8}$$

Integrating (4.8) from t to infinity along the optimal path, using the transversality condition $\lim_{\tau\to\infty} \mu(\tau) = 0$, gives

$$\mu(t) = -V^s(k(t), Q(t))e^{-\Gamma(t)}. \tag{4.9}$$

As in the competitive regime, the optimal policy corresponding to (4.1) is assumed unique (see footnote 4).

From (4.4)-(4.5) we see that (in an interior solution) the optimal allocation of x_1 is determined according to

$$z'(x_1) + \beta e'(Lx_1) = p_2, \tag{4.10}$$

where

$$\beta(t) = \frac{-\gamma(t)}{\lambda^s(t)} \tag{4.11}$$

is the negative of the shadow price of $Q(t)$, measured in consumption units.[6]

4.2 The Pigouvian hazard tax

While the competitive supply curve for the hazardous input is $Z'(X)$ (see Figure 1), the corresponding socially optimal curve is $Z'(X) + \beta e'(X)$ and the optimal allocation of X_1 is determined according to condition (4.10), as shown in Figure 2. Thus, $\beta(t)$ measures the unit cost of emission implied by the environmental threat, or the unit hazard cost.

Let X_1^s denote the demand for X_1 when emission is taxed at the rate β, i.e., X_1^s satisfies (4.10), rewritten in aggregate terms as

$$Z'(X_1^s) + \beta e'(X_1^s) = p_2, \tag{4.12}$$

[6]The hazardous stock Q, which advances the unwarranted occurrence, has a negative shadow price γ.

where it is recalled that $z(x) = Z(Lx)/L$. We now establish our main result:

Property: *Taxing emission at the rate $\beta(t)$ and redistributing the tax proceeds back as lump-sums implements the socially optimal allocation.*

Proof: Consider the household problem (3.13) when emission $e(X_1(t))$ is taxed at the rate $\beta(t)$, so that the total cost of the hazardous input is $Z(X_1(t))+\beta(t)e(X_1(t))$. Under this tax policy, the market allocation of the intermediate inputs agrees with the socially optimal allocation, determined according to (4.4)-(4.5), for any given capital stock k (see Figure 2). We need to show that this tax policy induces the households to make the socially optimal consumption-saving choices. Notice that the tax policy changes the per capita rent of the X_1-sector, defined in (3.9), to $[pX_1^s(k) - Z(X_1^s(k)) - \beta e(X_1^s(k))]/L$ while the lump-sum redistribution implies that the per capita tax proceeds $\beta e(X_1^s(k))/L$ are given back, hence the household's budget constraint (3.10) remains intact. Moreover, the competitive trajectories of $\lambda(t)$, $c(t)$ and $k(t)$ satisfy the necessary conditions (4.3) and (4.6), since these conditions are identical to (3.15)-(3.16), which (together with a transversality condition) determine the competitive allocation. Since the competitive and optimal allocations are unique, the household's consumption-saving decisions under the $\beta(t)$ policy must coincide with the socially optimal allocation.□

In view of this property we refer to $\beta(t)$ as the Pigouvian hazard tax. Implementing the regulation scheme requires the entire $\beta(t)$ process, which, except for some simple special cases, must resort to numerical methods. Much insight, however, can be gained by considering steady-state outcomes, to which we now turn.

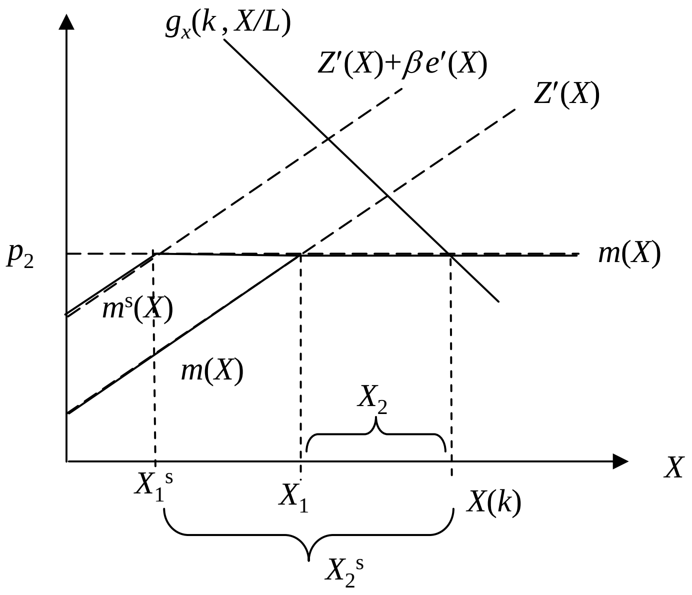

Figure 2: The competitive and social supply curves of X_1 are $Z'(X)$ and $Z'(X) + \beta e'(X)$, respectively. The competitive and social allocations of the intermediate inputs are X_j and X_j^s, $j = 1, 2$, respectively.

5 Steady states

A steady state occurs when k and Q, hence also X_1, X_2 and c, remain constant.[7] Whether the competitive or social policies converge to a steady state depends on the specification of the underlying functions (technology, utility, post-event value, emission and hazard rates) as well as on parameter values (δ, ρ, p_2). In this section we consider doomsday events (with a vanishing post-event value), assume convergence to a steady state and compare properties of the competitive and social steady states for a particular economy. To simplify notation, the labor force L is normalized at unity.

[7]Policy discussions regarding climate change presuppose a stabilized CO_2 concentration level, while debating what the stabilization target should be and the different ways to achieve it (Manne and Richels 1997, IPCC 2001, Pacala and Socolow 2004). In the present context, a stabilized CO_2 concentration corresponds to a constant stock Q.

5.1 Competitive steady state

Let the symbol '^' over a variable indicate its steady state value. From (2.1) and (3.10) we have

$$e(\hat{x}_1) = \delta\hat{Q} \tag{5.1}$$

(note that $x_1 = X_1$ under the $L = 1$ normalization) and

$$g(\hat{k}, \hat{x}) = p_2\hat{x}_2 + z(\hat{x}_1) + \hat{c}. \tag{5.2}$$

Differentiating (3.15) with respect to time, noting $\varphi \equiv 0$, $\dot{\hat{c}} = 0$ and $\hat{\Gamma} = (\rho + h(\hat{Q}))t$, and using (3.16) gives[8]

$$g_k(\hat{k}, \hat{x}) = \rho + h(\hat{Q}). \tag{5.3}$$

From (3.5) and (3.6) we obtain (for an interior solution with $x_2 > 0$)

$$z'(\hat{x}_1) = p_2 \tag{5.4}$$

and

$$g_x(\hat{k}, \hat{x}) = p_2. \tag{5.5}$$

Given the parameters δ, p_2 and ρ, and specifications of the functions $e(x)$, $z(x)$, $g(k, x)$ and $h(Q)$, equations (5.1)-(5.5) determine the five competitive steady-state variables $\hat{Q}$, $\hat{k}$, $\hat{c}$, $\hat{x}_1$ and $\hat{x} = \hat{x}_1 + \hat{x}_2$ in the following way: For an interior solution, (5.4) defines $\hat{x}_1 = X_1^c$ (see Figure 1) and (5.1) reduces to $\hat{Q} = e(X_1^c)/\delta$. Given $\hat{Q}$, (5.3) and (5.5) determine $\hat{k}$ and $\hat{x}$. Condition (5.2), then, defines $\hat{c}$.

[8]As expected, the interest rate on capital (cf. (3.2)) at a steady state equals the effective rate of discount.

5.2 Optimal steady state

Conditions (5.1), (5.2), (5.3) and (5.5) hold also in the socially optimal steady state (with the superscript s denoting socially optimal variables), while condition (5.4) is modified to

$$z'(\hat{x}_1^s) + \hat{\beta} e'(\hat{x}_1^s) = p_2. \tag{5.6}$$

We need another equation to determine the steady-state Pigouvian hazard tax rate $\hat{\beta}$.

At a steady state (4.9) reduces to $\hat{\mu} = -e^{-\hat{\Gamma}^s(t)} u(\hat{c}^s)/[\rho + h(\hat{Q}^s)]$, where it is recalled that $\hat{\Gamma}^s(t) = [\rho + h(\hat{Q}^s)]t$. With $\varphi = 0$, condition (4.7) reduces to $\dot{\hat{\gamma}} = \hat{\gamma}\delta + e^{-\hat{\Gamma}^s(t)} u(\hat{c}^s) h'(\hat{Q}^s)/[\rho + h(\hat{Q}^s)]$, which is readily integrated to give

$$\hat{\gamma} = \frac{-u(\hat{c}^s) h'(\hat{Q}^s)}{[\rho + h(\hat{Q}^s)][\rho + h(\hat{Q}^s) + \delta]} e^{-\hat{\Gamma}^s(t)},$$

where use has been made of the transversality condition implying that γ vanishes asymptotically. Condition (4.3) implies $\hat{\lambda}^s = u'(\hat{c}^s) e^{-\hat{\Gamma}^s(t)}$ and condition (4.11) defines the tax rate as $\hat{\beta} = -\hat{\gamma}/\hat{\lambda}^s$, hence

$$\hat{\beta} = \frac{u(\hat{c}^s) h'(\hat{Q}^s)}{u'(\hat{c}^s)[\rho + h(\hat{Q}^s)][\rho + h(\hat{Q}^s) + \delta]}. \tag{5.7}$$

Equations (5.1)-(5.3), (5.5)-(5.7) can be solved for the socially optimal steady-state variables: $\hat{Q}^s$, $\hat{k}^s$, $\hat{x}_1^s$, $\hat{x}^s$, $\hat{c}^s$ and $\hat{\beta}$.

5.3 Numerical illustration

To illustrate possible external effects associated with environmental threats, we compare the competitive and optimal steady states for the economy specified in Table 1. The capital and energy shares (α_k and α_x) are set to give a total share of 0.4 (leaving a labor share of 0.6). The intertemporal elasticity

σ and the utility discount rate ρ are set within the range given in Arrow et al. (2004). The constants u_0 and u_1 of the utility function $u(c) = u_0 \frac{c^{1-\sigma}}{1-\sigma} + u_1$ are conveniently chosen such that[9]

$$(i)\;\; u(\hat{c}) = \hat{c} \;\text{ and }\; (ii)\;\; \frac{u(\hat{c}^s)}{u'(\hat{c}^s)} = \hat{c}^s. \tag{5.8}$$

The competitive consumption $\hat{c}$ is independent of u while using (5.8)-(ii) in (5.7) allows solving for $\hat{c}^s$. Given $\hat{c}$ and $\hat{c}^s$, (5.8) identifies u_0 and u_1 (these are the values reported in Table 1).

Table 1: Specifications and parameter values

Function	Parameter	Value
$g(k,x) = \theta k^{\alpha_k} x^{\alpha_x}$	θ	0.6
	α_k	0.25
	α_x	0.15
$u(c) = u_0 \frac{c^{1-\sigma}}{1-\sigma} + u_1$	σ	2
	u_0	3.56457
	u_1	3.78679
Utility discount rate	ρ	0.005
$h(Q) = a(1 - e^{-bQ})$	a	0.01
	b	0.01
$z(x_1) = \xi x_1^2/2$	ξ	0.1
$e(x_1) = \varepsilon x_1^2/2$	ε	0.1
Environmental cleansing rate	δ	0.006

A condition such as (5.8)-(i) forms the basis for the Linearized Hamiltonian (Weitzman 2000), which allows to interpret the consumption rate $\hat{c}$ itself as the stationary-equivalent welfare measure corresponding to the value $v(\hat{k}, \hat{Q})$ of (3.13).[10] The socially optimal counterpart is $u(\hat{c}^s)$.

[9]Due to the state-dependent effective discount rate, the optimal policy is invariant with respect to the (positive) multiplicative constant u_0 but not with respect to the additive constant u_1. This property is in contrast to the event-free problem in which the optimal policy is invariant with respect to both parameters (see Weitzman 2000).

[10]See Tsur and Zemel's (2006) extension of Weitzman (1976) welfare measure to situations involving threats of abrupt events.

Table 2: Competitive and social steady state values

Variable	Symbol	Competitive	Social	Diff (%)
Capital	$\hat{k}$	71.81	105.87	47
Intermediate input	$\hat{x}$	1.927	2.160	12
Hazardous input	$\hat{x}_1$	1.5	0.3496	-77
Clean input	$\hat{x}_2$	0.4272	1.811	324
Hazard stock	$\hat{Q}$	18.75	1.0185	-95
Hazard rate	$h(\hat{Q})$	0.00171	0.000101	-94
Consumption	$\hat{c}$	1.751	1.883	7
Utility	$u(\hat{c})$	1.751	1.893	8
Value	$\hat{v},\ \hat{V}^s/L$	260.9	371.2	42
Pigouvian hazard tax	$\hat{\beta}$	0	3.29	

The steady state solutions are reported in Table 2. While the total intermediate input varies by only 12% between the competitive and social regimes, the allocations between the hazardous and clean inputs differ substantially (X_1 is reduced by 77% while X_2 increases more than four fold). The sharp reduction in the use of the hazardous input decreases the hazardous stock $\hat{Q}$ by 95% (from 18.75 under the competitive regime to only 1.02 in the social regime), which in turn lowers the hazard rate $h(\hat{Q})$ by 94% (from 0.0017 to 0.0001). This change in hazard implies, for example, that the probability of occurrence within the next ≈ 600 years under the competitive regime is the same as the probability that the event will occur within the next 10,000 years under the optimal policy[11] (no small consolation when dealing with doomsday events). Indeed, the reduced threat leads to an increase of 8% in the annuity-equivalent welfare (measured by the steady-state utility) and to a 42% increase in total welfare (measured by the steady-state value).

[11] At a steady state with constant hazard $\hat{h}$, the probability that the event will occur within the next n years is, noting (2.2), $1 - e^{-n\hat{h}}$.

The steady-state Pigouvian hazard tax is $\hat{\beta} = 3.29$. At this rate the tax cost of the hazardous input, $\hat{\beta}e(\hat{x}_1^s)$, amounts to more than 75% of its total cost: $\hat{\beta}e(\hat{x}_1^s)/[z(\hat{x}_1^s) + \hat{\beta}e(\hat{x}_1^s)] = 0.767$.

6 Conclusions

Environmental consequences of anthropogenic activities often entail threats of abrupt events in addition to gradual, tangible damages. As such environmental hazards typically bear public-bad features, they call for some form of regulation, which must be preemptive since it is too late to act after occurrence. The price of hazard generating activities, thus, should reflect also the cost associated with their contribution to advancing occurrence threats. In this work we identify this cost for a class of models, including greenhouse gas emissions whose atmospheric accumulation threatens to trigger climate-change related calamities, and offer a regulation mechanism based on this cost.

The proposed regulation employs a Pigouvian tax schedule that, when levied on hazardous emissions, implements the socially optimal allocation. We solve explicitly for the steady state of a particular schematic case to gain insight on possible effects of environmental hazard externality and the ensuing Pigouvian hazard tax rate.

The framework presented here is quite general and leaves a considerable room for extensions. Evidently, real-world applications will require refinement, such as in Nordhaus and Boyer (2000), with multiple emission sources, each contributing differently to the environmental threat. While population growth and exogenous technical change can be included with minor modifications, the incorporation of endogenous technical change is more challenging.

The latter extension requires the modeling of R&D efforts to reduce abatement costs or the cost of clean (backstop) technologies, as in Tsur and Zemel (2003, 2005). Such efforts consume resources that could otherwise serve the production of final goods, but at the same time reduce future environmental threats. The decision on the timing and extent of these R&D activities should account for these tradeoffs. Another possible extension allows to learn and continuously update estimates of the occurrence probability during the process. In this case one has to account also for the information content regarding the hazard associated with each feasible policy. While learning and expectations have been incorporated within economic models of gradual environmental damage (Karp and Zhang 2006), they are yet to be studied in the context of abrupt events.

References

Alley, R. B., Marotzke, J., Nordhaus, W. D., Overpeck, J. T., Peteet, D. M., Pielke Jr., R. S., Pierrehumbert, R. T., Rhines, P. B., Stocker, T. F., Talley, L. D. and Wallace, J. M.: 2003, Abrupt climate change, *Science* **299**, 2005–2010.

Arrow, K. J., Dasgupta, P., Goulder, L., Daily, G., Ehrlich, P., Heal, G., Levin, S., Mäler, K.-G., Schneider, S., Starrett, D. and Walker, B.: 2004, Are we consuming too much?, *Journal of Economic Perspectives* **18**, 147–172.

Bovenberg, A. L. and Goulder, L. H.: 1996, Optimal environmental taxation in the presence of other taxes: General-equilibrium analyses, *American Economic Review* **86**, 985–1000.

Brock, W. A. and Starrett, D.: 2003, Managing systems with non-convex positive feedback, *Environmental & Resource Economics* **26**, 575–602.

Broecker, W. S.: 1997, Thermohaline circulation, the Achilles heel of our climate system: Will man-made CO_2 upset the current balance?, *Science* **278**, 1582–1588.

Caputo, M. R.: 2005, *Foundations of Dynamic Economic Analysis*, Cambridge University Press.

Chakravorty, U., Roumasset, J. and Tse, K.: 1997, Endogenous substitution among energy resources and global warming, *Journal of Political Economy* **105**, 1201–1234.

Dasgupta, P. and Mäler, K.-G.: 2003, The economics of non-convex ecosystems: Introduction, *Environmental & Resource Economics* **26**, 499–525.

Goulder, L. H., Parry, I. W. H., Williams III, R. C. and Burtraw, D.: 1999, The cost-effectiveness of alternative instruments for environmental protection in a second-best setting, *Journal of Public Economics* **72**, 329–360.

IPCC: 2001, *Climate change 2001: Impacts, adaptation and vulnerability*, Cambridge University Press.

Karp, L. and Zhang, J.: 2006, Regulation with anticipated learning about environmental damages, *Journal of Environmental Economics & Management* **(forthcoming)**.

Mäler, K.-G.: 2000, Development, ecological resources and their management: A study of complex dynamic systems, *European Economic Review* **44**, 645–665.

Manne, A. and Richels, R.: 1997, On stabilizing CO_2 concentrations – cost-effective emission reduction strategies, *Environmental Modeling and Assessment* **2**, 251–265.

Nævdal, E.: 2006, Dynamic optimization in the presence of threshold effects when the location of the threshold is uncertain – with an application to a possible disintegration of the western antarctic ice sheet, *Journal of Economic Dynamics & Control* **(in press)**, doi:10.1016/j.jedc.2005.04.004.

Nordhaus, W. D.: 2001, Global warming economics, *Science* **294**, 1283–1284.

Nordhaus, W. D. and Boyer, J.: 2000, *Warming the World: Economic Models of Global Warming*, The MIT Press.

Pacala, S. W. and Socolow, R.: 2004, Stabilization wedges: Solving the climate problem for the next 50 years with current technologies, *Science* **305**, 968–972.

Tsur, Y. and Zemel, A.: 1996, Accounting for global warming risks: Resource management under event uncertainty, *Journal of Economic Dynamics & Control* **20**, 1289–1305.

Tsur, Y. and Zemel, A.: 1998, Pollution control in an uncertain environment, *Journal of Economic Dynamics & Control* **22**, 967–975.

Tsur, Y. and Zemel, A.: 2003, Optimal transition to backstop substitutes for nonrenewable resources, *Journal of Economic Dynamics & Control* **27**, 551–572.

Tsur, Y. and Zemel, A.: 2004, Endangered aquifers: Groundwater management under threats of catastrophic events, *Water Resources Research* **40**, 1–10.

Tsur, Y. and Zemel, A.: 2005, Scarcity, growth and R&D, *Journal of Environmental Economics & Management* **49**, 484–499.

Tsur, Y. and Zemel, A.: 2006, Welfare measurement under threats of environmental catastrophes, *Journal of Environmental Economics & Management* **(forthcoming)**.

Weitzman, M. L.: 1976, On the welfare significance of national product in a dynamic economy, *Quarterly Journal of Economics* **90**, 156–162.

Weitzman, M. L.: 2000, The linearised hamiltonian as comprehensive NDP, *Environment and Development Economics* **5**, 55–68.

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