Paper 5-3

AN ECONOMIC ANALYSIS OF DRAINAGE PROJECTS IN SINKING SOILS*

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INTRODUCTION

An area of approximately 40,000 dunams (10,000 acres) of swamps and a lake in the northern part of the Jordan Basin in upper Galilee was reclaimed in the mid 1950's when the first stage of the Hula Drainage Project was completed. The area has since been under cultivation. However, substantial parts of it suffer from winter floods and additional drainage projects are now being considered. The new project, now under planning and economic evaluation, is a complex system composed of several multistage subprojects. This paper develops the framework for the economic analysis of one of these subprojects, namely, the drainage of the peat soils area.

Peat soils form approximately one half of the drained area. These soils are very rich in organic materials—in some cases over 90% by volume—and cultivation created conditions favorable to their decomposition. This results in a gradual sinking of the soils which progresses faster in some parts of the valley than in others due to local conditions. The average rate is estimated to be in the order of 10 cm per annum. This loss of topographic elevation leads to an increase in the area which is lower than the winter

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level of water in the main drainage canals and is consequently subject to flooding. The lower the area, the higher the probability of winter floods and the damage to crops. It is expected that the sinking process will continue for several decades, lowering the area by several meters.

The sinking process can be controlled, to some extent, by special agricultural methods but these are considered expensive in terms of foregone income, and will probably not be used. On the other hand, the drainage canals, cutting through the area, can be deepened to prevent water from overflowing during the winter. This is the essence of the flood control projects now under consideration. Without going into technical details, we make the simplifying assumption that the larger the investment the deeper the canals and the smaller the flood damages.

The peat areas have been surveyed and maps prepared showing the available information on the composition of soil material. The sinking process can thus be forecast. We shall be able to estimate future floods with the existing drainage system or any new one.

The economic problem that emerges is that of determining optimum size and timing of the drainage project. Since the sinking process is gradual and large projects have to be built in stages—for technical and financial reasons—we shall discuss not only the optimum size and timing of a single project but also projects whose rate of construction is adapted to the rate of sinking of the peat soils. Therefore, the model developed is an investment process whose puvpose is to mitigate worsening economic conditions. One can take as additional examples the rate of construction of highways as a function of everincreasing congestion costs, or investment in advertising to remind the market of the existence of products which it otherwise slowly forgets [5].

As the foregoing discussion indicates, the investment projects are regarded as preventive measures and their contribution to the economy is a rising function of the damage or loss they prevent. This connects our analysis to Marglin's [3], who considered investment projects when demand for their product is rising. At this stage, our analysis is, like his, deterministic; which implies, for example, that we use expected values of the flood damages, instead of their distributions, or assume complete knowledge of the investment projects and their effects. It will become clear below that to some extent we also follow the model of capital accumulation developed by Eisner and Strotz [2]. The theoretical part of the article is general and applies to any case of capital accumulation with rising marginal product of capital. We prefer, however, to keep the discussion specific and to restrict it to the case of our particular flood control project. Generalization should follow easily.

The following section presents notation and our assumptions. Section 2 analyzes a single-stage drainage project, Section 3 deals with the multi-stage possibility. A continuous investment process is introduced in Section 4 and an application in Section 5.

1 Notations and Assumptions

Derivatives are indicated by primes, time derivatives by dots.

t calendar time;

r rate of interest.

The state of the area is characterized by the following variables (see Fig. 1):

A maximum potential income from the area (in dollars). In the present study, this is assumed to be independent of time. The assumption of a rising potential income can easily be incorporated [3].

$g(t)$ deterioration of income due to sinking. Since deterioration is a continuing process, we assume $\dot{g} \geqq 0$. Decomposition reduces the peat soil area, uncovering mineral soil. The area that sinks is thus diminishing. We assume, therefore, that $\ddot{g} \leqq 0$.

$A - g(t)$ actual income if no flood-control measures are taken. This value can become negative but then, unless drainage is improved, the area should probably be abandoned.

The project is constructed gradually, investment adding to its size. The flow of investment is, therefore, a measure of the rate of construction. The size of the project is measured in terms of accumulated investment. This creates a difficulty since the cost of construction will usually depend on the rate of investment. We shall distinguish between net and gross cost ([2], p. 471). Only the first is added to the project and can serve as a measure of its growth. This is the amount of "bricks" laid in the project, measured in money terms. The gross cost depends in addition on the rate of construction. This cost is the cost of laying the "bricks", including the value of the "bricks" themselves. It should be emphasized that the distinction drawn is artificial although the problem is real—very slow or very fast construction will generally be more expensive than investment at some optimum pace.*

* The optimum rate of construction depends on two components: (a) The sinking rate—the demand component, and (b) the cost of investment as a function of the rate of construction—the supply aspect.

Thus let

$\dot{w}(t)$ be the rate of construction (net cost) measured in dollars per unit of time (a year, say).
Without loss of generality we assume that we start from a zero size project, so that the size of the project at time t is

$$w(t) = \int_0^t \dot{w}(t)\, dt.$$

$\phi(\dot{w})$ is the gross cost of construction. As explained above, we assume that $\phi(\dot{w}) \geqq \dot{w}$.

The income of the area is a function of time and of the size of the project:

$P(t, w)$ income in dollars (per year).

In this work we assume (as did Marglin [3]) that the effect of the project can be expressed by a function $h(w)$, such that income is separable in the form

$$P(t, w) = A - g(t)\, h(w),$$

where

$h(w)$ is the effective flood control capacity of a project of size w and it is assumed that

$$0 \leqq h(w) \leqq 1, \quad h(0) = 1, \quad h'(w) < 0, \quad h''(w) > 0.$$

The assumptions on the signs of the first and second order derivatives of $h(w)$ are the usual production function assumptions. Engineers agree with these too, although in practice one may encounter regions of decreasing costs and it is not always easy to arrange subprojects in stages so that $h''(w) > 0$. The effect of the function $h(w)$ is illustrated graphically, for a special case, in Fig. 1.

We assume in the following that a flood-control project, once constructed, will last foreover. As service life of projects of this kind, if properly maintained, is very long, this seems a reasonable assumption. Maintenance costs are usually taken by engineers as a fixed percentage of investment outlays and as such they may be included in the construction costs and need not be treated separately.

Special notation is adopted for the discrete multi-stage cases:

t_i date of construction of stage $i(i = 1, 2, \ldots, n)$, $t_0 = 0$,

w_i size of project after the construction of stage i,

x_i investment of stage i, so that $w_i = \sum_{k=1}^{i} x_k$.

For simplicity, we assume a gross cost function of the form $x_i + c$ (where c is fixed cost per stage) for discrete cases.

2 A Single-Stage Project

Valuable insight is gained by starting the discussion with a single stage case. A single stage project of size w_1 will be constructed at time t_1. Present value of net income from the area is given by

$$y = \int_0^{t_1} [A - g(t)]\, e^{-rt}\, dt + \int_{t_1}^{\infty} [A - g(t)\, h(w_1)]\, e^{-rt}\, dt - (x_1 + c)\, e^{-rt_1}. \quad (1)$$

Note that $w_1 = x_1$.

y in (1) is to be maximized with respect to t_1 and to w_1. Since A is the maximum annual income, y is bounded for positive r. The necessary conditions for optimum timing and size are $\partial y/\partial t_1 = \partial y/\partial w_1 = 0$. Second order conditions can be shown to hold.

$$\frac{\partial y}{\partial t_1} = 0 \rightarrow r(x_1 + c) = g(t_1)\,[1 - h(w_1)]. \quad (2)$$

That is, investment will take place when the (annual) interest cost will be equal to the (annual) value of the damage prevented.

$$\frac{\partial y}{\partial w_1} = 0 \rightarrow \int_{t_1}^{\infty} g(t)\, h'(w_1)\, e^{-rt}\, dt = -e^{-rt_1}. \quad (3)$$

The integrand in (3) is the annual value of the damage prevented by the marginal dollar. The integral is thus the marginal value of the investment. It equals, at the optimum, \$ 1 discounted from t_1.

There still remains the question whether to build or not and for this purpose it will be useful to define:

$$D \equiv \int_0^{t_1} [A - g(t)]\, e^{-rt}\, dt;$$

$$E \equiv \int_{t_1}^{\infty} [A - g(t)]\, e^{-rt}\, dt;$$

$$F \equiv \int_{t_1}^{\infty} [A - g(t)\, h(w_1)]\, e^{-rt}\, dt;$$

$$G \equiv (x_1 + c)\, e^{-rt_1}.$$

The economic rent of the project R is the value of the damage prevented.

$$R = F - E - G = \int_{t_1}^{\infty} \{g(t)\,[1 - h(w_1)] - r(x_1 + c)\}\, e^{-rt}\, dt. \quad (4)$$

Two cases can be distinguished. In one of them—perhaps the flooding of residential areas—the project should be constructed whenever the rent, R, is positive. This will happen if in the solution of (2) and (3) $0 < t_1 < \infty$, since by (2) the integrand in (4) is zero for $t = t_1$ and non-negative for $t > t_1$, since $\dot{g} \geqq 0$. However, in our case there exists the alternative of abandoning the area. Here the criterion for construction should be $F - G > 0$ (note that E may be negative). If $D < 0$, the area will not be cultivated until the completion of the project at t_1.

Construction may have to start immediately (perhaps for political reasons); optimum size is then determined by solving (3) for $t_1 = 0$. Similarly if the solution of (2) and (3) yields $t_1 \leqq 0$ (in this case $g(t)$ should be defined for negative values of t), construction should be immediate and of the same size as if $t_1 = 0$ was forced.

3 Multi-Stage Projects

If division is possible, construction in stages may increase the efficiency of the system. Net income from an n-stage project is

$$y = \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} [A - g(t)\, h(w_i)]\, e^{-rt}\, dt + \int_{t_n}^{\infty} [A - g(t)\, h(w_n)]\, e^{-rt}\, dt$$
$$- \sum_{n=1}^{n} (x_i + c)\, e^{-rt_i}. \quad (5)$$

For convenience we define here $w_0 = 0$.

Remember, $w_i - w_{i-1} = x_i$ $(i = 1, ..., n)$, $h(0) = 1$, $t_0 = 0$.

Again,

$$\frac{\partial y}{\partial t_i} = 0 \rightarrow g(t_i)\,[h(w_{i-1}) - h(w_i)] = r(x_i + c) \qquad (i = 1, 2, ..., n). \tag{6}$$

(Since $t_0 = 0$, this variable cannot be included among the parameters of maximization.)

$$\frac{\partial y}{\partial w_i} = 0 \rightarrow \int_{t_i}^{t_{i+1}} g(t)\,h'(w_i)\,e^{-rt}\,dt = e^{-rt_{i+1}} - e^{-rt_i} \qquad (i = 1, 2, ..., n-1). \tag{7a}$$

$$\frac{\partial y}{\partial w_n} = 0 \rightarrow \int_{t_n}^{\infty} g(t)\,h'(w_n)\,e^{-rt}\,dt = -e^{-rt_n}. \tag{7b}$$

The system (6) and (7) is a set of simultaneous equations. In practice one may encounter cases which will make their "step-wise" solution possible. Some examples will illustrate this point.

a) Assume that the size of the stages is predetermined (this will be the situation in the application illustrated below). Then optimum timing is determined by (6), starting from t_1. Equation (6) may be written in the more general form

$$P(t_i, w_i) - P(t_i, w_{i-1}) = r(x_i + c), \tag{6'}$$

which emphasizes that a stage will be added to the project when the additional income due to the prevention of damage is equal to the interest cost of the capital invested at this stage.

b) In another case, the sequence $\{t_i\}$ may be predetermined, perhaps in the form $t_i = i$, or by any other pattern. Then the set (6) is void and (7) can be solved equation after equation, from w_1 to w_n.

Equations (6) and (7) show that the optimum size of the project at point t_i depends, in general, on the planning horizon. It is instructive to note that when either $\{w_i\}$ or $\{t_i\}$ is predetermined, the optimum size or timing of investment is independent of the planning horizon.*

* This conclusion holds only for linear cost functions and not for the general function $\varphi(w)$.

An intuitive explanation for this is connected to the fact that a project of size w_i contributes by preventing damage during period (t_i, t_{i+1}) and also "delivers" a project of size w_i at t_{i+1}.

c) Another interesting case might be the one in which only the date of completion of the project, t_n, is predetermined. Then one can go "backwards" from t_n, first determining w_n then w_{n-1}, t_{n-1}, etc. This method of solution as well as the previous ones can be interpreted as a dynamic programming algorithm [1]. The recurrence relation for the present case (c) is

$$f(t_i, w_i) = \max_{t_i, w_i} \left\{ \int_{t_i}^{t_{i+1}} [A - g(t)\, h(w_i)]\, e^{-rt}\, dt - (x_i + c)\, e^{-rt_i} + f(t_{i+1}, w_{i+1}) \right\}, \tag{8}$$

where $f(t_i, w_i)$ is the maximum present value of income if the multi-stage project starts at t_i, and is constructed in $n - i$ stages.

Dynamic programming can be applied to the numerical solution of the system (6) and (7) even if these equations must be solved simultaneously and not step-wise in the sense of points (a)–(c) above.

If the date of the final stage is predetermined, the number of stages is dictated by the solution. If, on the other hand, n is given exogeneously, one could search for the corresponding t_n.

Consider the simplest of the multi-stage projects—the two-stage case. The single-stage project of Section 2 can be obtained as the limit of the two-stage project as $t_2 \to \infty$. Thus, if the solution to the maximization of income from the two-stage project yields $t_2 < \infty$, income from this project will be larger than income from the single-stage case. This can be generalized to the multi-stage case.

We may consider a multi-stage process with an infinite number of stages. Then (6) and (7), expressing the necessary conditions for optimal investment, will form infinite sets of equations.

4 Continuous Construction

Within the context of flood control projects, a continuous construction model is perhaps only of theoretical interest. However, it will be an approximate description of a multi-stage discrete model with small intervals between the stages. The solution of the continuous investment case is concise and one may wish to calculate it to gain more insight into the solution of discrete models. In other cases (consider advertising) it may be a closer description of reality then the discrete model.

In the continuous case, we do not speak of fixed costs, c, as in the discrete case, but permit outlays associated with construction to be larger than net investment and depend on the rate of investment. Thus $\phi(\dot{w}) \geqq \dot{w}$. We start, however, with the case $\phi(\dot{w}) = \dot{w}$ and mention the more general, and complicated, case later.

Present value of net income, if $\phi(\dot{w}) = \dot{w}$, is

$$y = \int_0^\infty [A - g(t)\, h(w) - \dot{w}(t)]\, e^{-rt}\, dt. \tag{9}$$

Maximizing y in (9), we use the calculus of variation ([1], p. 40). Let H stand for the integrand in (9), then by the Euler–Lagrange equation

$$\frac{\partial H}{\partial w} - \frac{d}{dt}\frac{\partial H}{\partial \dot{w}} = 0,$$

we obtain

$$h'(w) = \frac{-r}{g(t)}. \tag{10}$$

The end point condition reduces in this case to

$$\lim_{t \to \infty} e^{-rt} = 0,$$

which is automatically satisfied.

From (10)—since $h'(w)$ is a monotonic function—one can deduce the rate of investment $\dot{w}(t)$, once the explicit forms of the functions $g(t)$ and $h(w)$ are given. Equation (10) thus indicates the optimum path of the project's future history.

Some further observations are noted below:

a) Condition (10) can also be obtained from (6)—the first order condition for optimum timing in the discrete case—which can be rewritten as (remember that $c = 0$)

$$g(t_i)\,\frac{h(w_{i-1}) - h(w_i)}{w_{i-1} - w_i} = -r. \tag{11}$$

Taking the limit of (11) as $w_{i-1} \to w_i$, we get (10). For a similar approach in the context of dynamic programming see ([4], p. 231).

b) The optimum initial size of the project, w_0 at $t_0 = 0$, is given by (10) and it is such that

$$h'(w_0) = -\frac{r}{g(t_0)}.$$

Thus, the process will start with an initial investment of w_0 and then continue in the path dictated by (10).*†

c) It is important to remember that we found in this and other sections the conditions for maximum net income or minimum losses. Denoting by y^* the value of the integral in (9) when investment follows the optimum path dictated by (10), the project will be economically justified only if $y^* - w_0 \geqq 0$.

Note also that the element of the construction cost in (9) is

$$\int_0^\infty \dot{w}e^{-rt}\,dt = -w_0 + \int_0^\infty (rw)\,e^{-rt}\,dt. \tag{12}$$

The right hand side of (12), obtained by integration by parts, is the difference between the service cost of capital invested in the project and the initial investment, w_0.

d) Differentiating (10) with respect to time one gets

$$\dot{w} = \frac{r\dot{g}}{[g(t)]^2\,h''(w)}. \tag{13}$$

By assumption $\dot{g} \geqq 0$, $h''(w) > 0$. So long as $h''(w) < \infty$ and $\dot{g} > 0$ we have $\dot{w} > 0$. That is, construction will proceed continuously. However, it will stop when $\dot{g} = 0$.

The result, stating that $\dot{w} \geqq 0$, is welcome, since the project cannot be scrapped at a price, disinvestment—that is $\dot{w} < 0$, is meaningless.

e) In general, income from the area will not be constant. Differentiating $A - g(t)\,h(w)$ with respect to time, assuming (10), we obtain the rate of change of income along the optimum path

$$\frac{d[A - g(t)\,h(w)]}{dt} = r\dot{w} - \dot{g}h(w). \tag{14}$$

It is not clear what the sign of (14) is.

* Note that initial adjustment is here instantaneous. This is due to the assumption of $\phi(\dot{w}) = \dot{w}$ (compare with Eisner and Strotz [2]).

† Remember that we do not assume that a project of any size exists beforehand. This point can easily be modified.

f) Part of the foregoing discussion indicates that this is a somewhat degenerate case. Due to the linearity of the cost function in (9), the derivative $\dot{w}$ does not appear in (10), and there is only one optimum path of investment (see also point (*b*) in Section 3).

In the more general case, where $\phi(\dot{w})$ is not a linear function of $\dot{w}$, the present value of the income is

$$y = \int_0^\infty [A - g(t)\, h(w) - \phi(\dot{w})]\, e^{-rt}\, dt. \tag{15}$$

The necessary condition for optimum path is

$$-\phi''(\dot{w})\, \ddot{w} + r\phi'(\dot{w}) + g(t)\, h'(w) = 0, \tag{16}$$

with the end condition

$$\lim_{t\to\infty} \phi'(\dot{w})\, e^{-rt} = 0. \tag{17}$$

Further investigation of Eq. (16) has been deferred to a later work.

5 An Example

The example presented in this section is based on preliminary data from the Hula project and on some arbitrary assumptions. The analysis should not be taken as a recommendation of any sort.

The planned flood control project is divided into five stages (see Table 1). The first stage, if constructed, will reduce the expected flooded area in 1969 from 4,999 dn to 1,589 dn. Cost of construction is* IL 1,540,000 or IL 452 per dunam. Stage 2, if carried out in 1969, will reduce the expected flooded area by 883 dn in that year, at a cost of IL 2,264 per dunam. The marginal cost increases from stage to stage. This is consistent with our assumption of $h''(w) > 0$.

We assume a rate of interest of 10% (8% capital cost and 2% maintenance). At this rate, the present value of a dunam of land "saved" from the floods (in terms of expected value) is IL 1,282. Thus, according to the last column of Table 1, only stage 1 should be constructed in 1969.

Information similar to that given in Table 1 was projected for the period 1969–2000 from technical data. Thus we could estimate future values of the

* IL 3.5 = $ 1; 1 dn = 0.25 acres.

TABLE 1 PROJECT DESCRIPTION (1969)

Stage	Stage Identification in Hula Project	Cost of Project (w_i) (IL '000)	Cost of Stage (x_i) (IL '000)	Expected Value of Flooded Area (dn)	Change in Area Flooded (dn)	Marginal Cost (IL/dn)
0	Present state			4,999		
1	59.15	1,540	1,540	1,589	3,410	452
2	58.65	3,539	1,999	706	883	2,264
3	58.65 − 0.5	4,963	1,424	570	136	10,471
4	58.65 − 0.5+	7,888	2,925	363	207	14,130
5	58.65 − 0.5++	11,110	3,222	303	60	53,700

Notes:
Costs are based on 1969 data;
Project's effect, in terms of area flooded, is for 1969;
1 dunam = 0.25 acres;
Fixed costs $c = 0$.
$ 1 = IL 3.50.

functions $g(t)$ and $g(t)\,h(w)$. At this point, Eq. (6) was utilized to calculate optimum t_i values. This analysis is carried out in Table 2. Potential income from the project area is IL 6,691,000 per annum. If the project is not carried out, the damage in 1969 will be IL 640,000. Construction of stage 1 in 1969 will contribute IL 437,000 of damage prevention at an interest cost of IL 154,000. It should therefore be constructed immediately.

Stage 2 is to be constructed in 1977. This is the first year in which the annual value of the damage prevented by stage 2 will be higher than the interest cost on the investment at this stage. Stage 3 will be constructed in 1987. The calculations were followed up to the year 2000, showing that stage 4 will not be constructed in this period. The resulting income flows were plotted in Fig. 1.

6 Concluding Remarks

This paper has presented a theoretical framework for the analysis of flood-control projects in the Hula peat soils, and, we trust, for some other cases as well. It serves as a starting point for further research and as a guide to the empirical work which is now in progress.

TABLE 2: CONSTRUCTION PROCESS (IN THOUSANDS OF ISRAELI POUNDS)

Year (t)	1969	1977	1987	2000
Present state—no construction				
Net income $[A - g(t)]$	6,051	5,557	4,852	4,129
Damage $[g(t)]$	640	1,134	1,839	2,562
Stage 1				
Net income $[A - g(t)\,h(w_1)]$	6,488	6,301		
Damage $[g(t)\,h(w_1)]$	203	390		
Damage prevented if stage constructed	437			
Interest cost (rx_1)	154			
Stage 2				
Net income $[A - g(t)\,h(w_2)]$		6,500	5,968	
Damage $[g(t)\,h(w_2)]$		191	723	
Damage prevented if stage constructed		199		
Interest cost (rx_2)		200		
Stage 3				
Net income $[A - g(t)\,h(w_3)]$			6,109	5,075
Damage $[g(t)\,h(w_3)]$			582	1,616
Damage prevented if stage constructed			141	
Interest cost (rx_3)			142	
Stage 4				
Net income $[A - g(t)\,h(w_4)]$				5,079
Damage $[g(t)\,h(w_4)]$				1,612
Damage prevented if stage constructed				4
Interest cost (rx_4)				293
Optimum size of project (w_i)	1,540	3,539	4,963	4,963

Notes:

Potential income: $A =$ IL 6,691,000;

Damage prevented: $P(t_i, w_i) - P(t_i, w_{i-1}) = g(t_i)\,[h(w_{i-1}) - h(w_i)]$;

Column headings show construction dates, except for 2000;

A rate of interest $r = 0.10$ is assumed.

Further work in this study will be in three directions: a) The integration of the analysis of the peat soils project with the analysis of the rest of the Hula Basin drainage system; b) The incorporation of elements of uncertainty and accumulated information in the analysis; c) Extension of the analysis of Section 4 to a more general cost function.

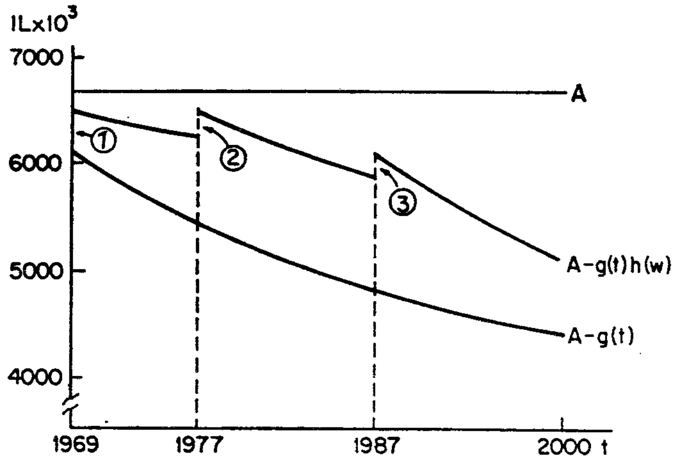

Fig. 1. Future income flows in project area

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