Intergenerational Transfers of Farmers: The Israeli Experience

Yoav Kislev

This paper is a compilation of Chapters 4-9 from *Intergenerational Transfers of Farmers in Different Institutional Environments*, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal Kimhi, and Ramon Lopez, 1995. Ayal Kimhi selected the chapters based on Kislev's contribution.

4. The Basic Approach to the Farm Transfer in the Moshav

Farm transfer, particularly in the moshav, is a complex process encompassing economic, social, institutional and psychological issues. Its study is difficult because a lot occurs within the family and is only rarely visible to the outside observer. We are attempting in this chapter to outline a theoretical and conceptual framework which will facilitate the analysis of the process of transfer with its multidimensional facets. Wherever possible, the discussion is connected to research done in this BARD project and included in the present Report.

Biology and the Family¹

With genetic variability and inheritance, evolution develops traits that increase the probability of survival and reproduction of the species. Thus parents, particularly mothers, are altruistic towards their youngsters, they feed, shelter and train them. But once the youngsters have grown up and can fend for themselves, they are rejected by their parents--they are even chased away from the territory-freeing space and parental energy for the next batch of offsprings. Similarly, youngsters in the animal world do not exhibit altruism towards their aging parents; such a trait has no advantage in the competitive evolutionary selection process.

Human families are undoubtedly driven by biological forces-- mother's love and affection are genetic traits--but they are also controlled by economic and social considerations. We see a certain degree of "altruism" of children toward their parents, and lesser rejection of grown-up offsprings and a smaller degree of parents-children separation than we might have expected if family behavior was dictated by biology alone. This pattern is characteristic of societies in which the extended family is typical and is seen also in some modern, nuclear families. Co-habitation of adult children and parents is the prevailing mode in farm-families in Israel. In suggesting his theory of "gluing the generations,"

¹ This section draws on Shubik (1981).

Shubik (1981) constructed a three stage model of life cycle. With the possibility of adult cohabitation, a four-stage model is called for: 1. Childhood, fully supported; 2. Young adults living with parents; 3. Adults, perhaps supporting their parents; 4. Old age. Family members are productive and earn income in stages 2 and 3, and are unproductive in stages 1 and 4. Children are cared for by their parents and old people live off their savings or are supported by their stage 3 children.

Viewing life cycle as a four-stage process raises the behavioral questions of the determinants of the number of children, the intensity of their care (investment in children), division of work and consumption in the cohabitation stage, saving for old age, support of old people, inheritance, and many more. Quite a few of these questions were addressed already, particularly by Gary Becker (1991) and his followers, but several puzzles still remained for us to work on.

Farm Transfer in the Moshav

The moshav is a cooperative village of typically 60-100 family farms. Cooperation in most moshavim (pl. of moshav) is today much less intensive than it was in the past when members purchased all inputs and marketed the farm products through the cooperative association which also served as the village financial intermediary (Zusman 1988). Cooperation collapsed in wake of a severe financial crisis, which was triggered by anti-inflationary policies of tight money implemented in Israel in 1985-86. But the roots of the crisis were deeper: with financial cooperation and government support, moral hazard behavior developed, moshavim and second-order cooperatives overexpanded, and they collapsed when the economic environment changed for the worse (Kislev, Lerman, and Zusman 1991).

Nowadays, most families in the moshav operate their farms independently, but some economic and legal ties still bind the members in the village community. In particular, land in the moshavim is not private but nationally owned. Each family received through the moshav a plot to cultivate. This homestead cannot be divided. Members may sell their share in the cooperative together with the right to the land and dwelling on their farm, but the newcomer must be accepted to membership in the cooperative association. In this way the association has a veto power to prevent the selling of a farm to a party which is not to its liking. Consequently, a farm in a moshav cannot be acquired by a bank or by another institutional lender. This limits significantly the financial freedom of the members who cannot use their farm as collateral in the credit market.

Within-family farm transfer is also constrained by rules and regulations. Parents may decide on one adult child as the "continuing son" (or daughter), mostly it will be done when the child is already married and the spouse is also willing to live in the village. Once this decision has been made, the moshav usually accepts the young candidate for membership in the cooperative association. Only then can the young couple build a second house on the family plot.

In many cases the succeeding couple joins the parents on the farm, quite often they work the farm together and share its income. Stage 2 of the life cycle thus begins. As the parents get older, their ability to work on the farm or off it is reduced and they enter Stage 4 in their life cycle in which they

live off their savings or draw on farm income which means now that they are supported by the younger couple operating the farm.

The "continuing son" is not automatically the owner of the farm. Legal ownership transfer is a separate stage, sometimes parents transfer the farm legally to the young couple in their life--but most often only after several years of cohabitation--and in other cases ownership is acquired by inheritance.

Delayed legal transfer facilitates testing of the ability of the young couple to carry farm work and to live together with the parents, but it also introduces a severe difficulty. Without a will specifying the bequest of the farm to the succeeding family, the property of the parents is subject to division between the heirs; though the farm itself cannot be subdivided. A will specifies the bequest, and a will can be modified up to the last day of a person's life. Thus the young family can find after many years of living on the farm and working it that the farm was actually bequeathed to someone else. Agreements and contracts made before the will was opened are not binding in face of a contradictory will. Thus, so long as the farm was not transferred to them legally, the young farm operators live with uncertainty. We analyze one aspect of this uncertainty and the nature of the parents-child relation in Chapter 8 (Transfer of Ownership).

Our analytical challenge was to construct a theoretical framework for the analysis of the various economic and social aspects of farm transfer in the moshav. We borrowed most of our analytical instruments from Becker (1991), but departed from his theory wherever the particulars of our problem so dictated. But before reporting on the analytical and econometric research of the project in the chapters that follow, we present two background studies. Shuky Regev (1995) summarizes in Chapter 2 the legal aspects of farm transfer in the moshav in a more detailed and accurate way than could be done in this introductory chapter, and Naomi Nevo (1995) reports on an ethnographical study of farm succession in several moshavim representing much of the diversity of social and economic life in cooperative villages in Israel.

Children on the Farm

Children on the farm are provided for by their parents, just like children anywhere else. They often contribute work, but this does not change the analysis much. It can be seen as a reduction of the cost of raising children on the farm compared to non-farm situations.

A particular problem farm parents face is the decision on the succeeding child. Investing in their children, they may mark children to non-farm life and assist them with schooling and their first steps in the urban settings. The succeeding child can be expected to receive less schooling and, in preparation for future farm life, this son or daughter may be expected to take vocational education or university training in agriculture. Ayal Kimhi (1995a) analyzes in Chapter 10 an overlapping-generations model of occupational choice and shows that comparative advantage in accumulation of sector-specific human capital may affect the choice of successor. Two factors may mitigate early specialization of farm children and its reflection in their education: a. schooling at all levels is relatively inexpensive in Israel; and b. at least two thirds of the farm operators in the moshavim are

not full time farmers and they draw substantial shares of their income off their farms. One may expect therefore that children's schooling on farms will depend more on family tradition, income, social status, and norms than on the anticipation of the future career.

A particular problem is raised by timing. We elaborate below on the question of synchronization of the farm transfer with the parent's age and labor productivity. Often when the first child is ready to accept the farm, the parents are too young to relinquish it. The farm may also be too small then to support both the parent-family still raising younger children and the young family about to grow soon. On the other hand, if the last child is to receive the farm, the parents may be too old by the time the young family steps in. Moreover, the parents run the danger that the last child will not wish to join them on the farm.

The Succeeding Child and Other Siblings

The succeeding child is to receive the farm. With it may come an obligation to compensate the other brothers and sisters and to care for the parents in their old age. The compensation may be explicit and agreed upon in advance--contracts signed with legal professional assistance are common--or it may be implicit. It often takes the form of the parents drawing on the farm's income to support other children setting up households elsewhere. The magnitude of the obligation of caring for the parents in their old age is not known in advance. Some people die relatively young and healthy and others have to be supported intensively for many years. The possibility of intra-family old age insurance and its implication for the value of the assets transferred between parents and children is taken up by Claudio Pesquin (1995) in Chapter 17 in an analysis based on the model developed by Kotlikoff and Spivak (1981).

Whatever the formal or informal arrangement between the parents and the succeeding family when joining the farm or when the legal transfer is materialized, the young family faces a significant degree of uncertainty as to the expenses they will have to carry. This uncertainty can be largely reduced if the family divides its wealth completely at an early stage, each child receiving whatever is due him or her and the parents receive their share. If this is actually done, uncertainty is indeed reduced, but with it the family loses in mutual insurance. Moreover, since future needs of the children and the parents are uncertain, the division of the family property may be difficult, families are even reluctant to raise these issues in internal discussions.

The possibility that some of the farm income and wealth will have to be shared with other family members and the uncertainty associated with this eventuality create agency problems. The young couple may work less and invest less on the farm than if it were wholly theirs. The parents, who may be uncertain about the support they could expect from the succeeding couple if the need arises, may try to share more of its wealth with the other children thus hedging by diversifying their saving and old age insurance portfolio. This problem is taken up in Chapter 8.

Timing Problems

It was already pointed out that timing affects the choice of the succeeding child. With primogeniture, the farm may be burdened by two families when their combined needs are too heavy to be supported. A delayed decision increases the risk that the parents will not find a child willing to settle on the farm and take it over from them when the time arrives.

Another consideration is also involved here. The productivity of the farm operator, both as a laborer and as a manager, increases with age and experience for 20-30 years but eventually it declines sharply. Synchronization of father-son work on the farm and optimal timing of transfer can increase the stream of income from the farm and its wealth. Ayal Kimhi (1995b) reports in Chapter 18 age-productivity profile of farm operators.

Still another problem is the decision on the responsibility for running the farm. When should the experienced father relinquish his decision power in favor of the less experienced but more dynamic son who may also be inclined to take risks his father can judge excessive?

Again another timing problem is the question of the legal transfer of the farm. Parents keep the farm as a property registered on their name even after they ceased contributing significantly labor or management to its running. In this way they insure against the possibility that the young couple will disappoint them and fail to support them adequately at their old age. For the same and symmetric reasons, the young couple resents working the farm which is not registered to them. This situation worries particularly wives of succeeding sons. They work side by side with their husbands in creating a home and accumulating farm wealth and may one day be dislodged at the stroke of a pen. They will be particularly disadvantaged in cases of divorce or their husbands' early death. Thus timing of legal transfer is a painful question for many farm families. It is again one of the questions families hesitate to raise even in the privacy of their own homes. The question is discussed analytically in Chapter 8 on Transfer of Ownership.

Provision for Retirement

Theoretically, the retiring parents can rely on the farm and on contracts they may formulate--explicitly or implicitly--with the succeeding couple to furnish them with their retirement needs. But sole reliance on future farm income entails great risks, uncertainties for both parties, and difficulties of recontracting whenever the circumstances change. It is therefore in the interest of both sides that some retirement funds be provided, even if the returns on such funds are lower than the returns on investment in the farm and its assets.

Analytically we are facing the following problem: the extended family of the young and the old provides insurance to both sides (the parents can also reduce their standard of living in case the succeeding couple runs into unexpected economic difficulties), but the realization of the payment of the "compensation" may be hard. People are willing to forego some of the insurance benefits for smoother financial streams and easier internal family relations. Particularly, the need of one adult side

to ask permission to execute expenses may be unaccepted. This permanent reliance on the other side often reduces the welfare of the dependent side.

The problem was depicted here in terms of the need of parents to provide for their retirement even if this action reduces the total farm wealth. But we may start from the other end of the spectrum of possibilities: if not for the farm, parents would have had strong incentives to save in commercial financial institutions. Knowing that they can draw on the farm, they may be inclined to reduce their outside saving. In this way, it may seem to the succeeding couple that the parents draw "a great share" of the farm income while, actually, the parents do so because they invested comparatively large resources in the farm and not outside. They have accumulated assets which now serve the two families in cohabitation on the farm.

The Analytic Approach

Two strands of analysis are now common in the literature dealing with the economic problems of families. The one, today already in the status of the traditional, perhaps classic, approach is Becker's (1991) framework in which altruism is the ruling force, particularly in considering the behavior of parents toward their children. Formally, the approach materializes in models in which the children's welfare appears as an argument in the utility function of their parents. The second approach treats family relations as a sets of conflicts leading to bargaining and ending in contractual arrangements. Here the analytical tools are borrowed from game theory.

Writers in the second tradition commonly criticize Becker's approach as being naive, and offer their view of the family as a nexus of conflicting interests as more realistic. We suggest here that the two approaches are complementary rather than competitive. So long as the children are small and depend on their parents, it may be more appropriate to view the parents-children relationships as dictated by altruism, but once they are grown, and establish their own families--conflicts may arise. We therefore view the alternative analytical approaches as appropriately applied in different stages of the family life cycle. Accordingly, Chapter 5 "A Single Utility Function" is written in the Becker tradition and discusses the relations of parents and young children; Chapter 7 "Sharing Farm Work" discusses potential contractual arrangement between the parents and the succeeding child when working the farm together.

5. A Single Utility Function

In this chapter we present a theoretical analysis of the farm family and its wealth transfer under the assumption that the family's behavior can be described as guided by a single utility function with incomes of the family members as arguments. This is the assumption adopted by Becker in the *Treatise* (1991), and we shall follow that analysis; particularly we draw on Becker's Chapter 6.

The assumption of a single utility function applies well to the early stages in the family life cycle, when the children are still young and depend on their parents who are then responsible for the children's day to day care and provide for their future. Adopting a single utility function, intra-family conflicts are assumed away. At this stage, when the family is still young, potential disagreements are between husband and wife, and we disregard them when focusing on inter-generational issues.

The model of the family as a single utility function is often described as altruistic: the parents care for their children and are ready to sacrifice their own welfare for the welfare of their offsprings. Clearly parents are altruistic, but there is more to this model than just altruism. Particularly, economic resources of all members of the family are pooled and the family--by the decision of the parents-divides these resources among all members. Becker derives important behavioral conclusions within this framework.

We expand Becker's analysis to include issues of particular interest to farm transfer. In general, we rely on the findings in the <u>Treatise</u>, but there are also some repetitions--where it was felt that they were necessary for completion.

The Basic Two-Period Model

There are only two points in time, period t and period t+1, and the distinction between flows and stocks is not maintained: we often talk interchangeably of income and capital, or wealth. In period t children live with their parents, who provide for the children from their income. In Becker's analysis the parents disappear between period t and t+1 and in the second period there are only grown-up children who now earn income of their own. The parents divide their wealth in period t between current consumption and capital they leave to their children for the second period. The children's income in the second period consists of the returns to the capital they received from the parents plus their own endowment and luck. In the first stages of the analysis, parents know in advance the children's endowment and future luck and allocate capital accordingly. Mostly, parents are assumed to be neutral toward their children: they do not have preference for any particular child or gender.

Saving for Old Age

Most farmers expect to live after the transfer of their assets to the succeeding children. We therefore start our analysis by adding to Becker's model future consumption of the parents.

At this stage, we assume a complete separation in period t+1 between the children's income and that of the parents. Even if they live together on the same farm, only the succeeding family draws on the farm's income for consumption; the parents draw on their savings. Let the consumption of the parents in the periods be Z_t and Z_{t+1} . For the time being, there is one child whose consumption in period t is included with the parents' in the term Z_t and whose wealth in period t+1 is marked I_{t+1}. The parents' utility function in t is

$$U = U(Z_t, Z_{t+1}, I_{t+1})$$
(1)

To prepare for the future, the parents save in their own "account" Y_t , and provide the child with y_t unit of capital. Capital is measured in efficiency units of identical returns. Physically, capital units may be years of schooling, cows, or IBM shares; but in this analysis they are all measured by the same efficiency unit. Each unit of y_t yields at t+1 w_{t+1} "dollars." The cost of a unit of y is π . Hence, the rate of return on investments in children, r_t , is implicitly defined by $\pi = W_{t+1}/(1+r_t)$. If parents have access to the saving market and if investments in children are non-zero, all rates of return have to be equalized. Hence, if D=1+r_t, then:

$$Y_t = Z_{t+1}/D \tag{2}$$

Parents' income is It and the budget (or wealth) constraint in the first period is therefore

$$Z_t + Y_t + \pi_t y_t = I_t \tag{3}$$

The parents contribute only part of the child's wealth (earning ability); in addition to y_t , a child is also attributed with own endowment, e_{t+1} , and luck, u_{t+1} , which for the time being are assumed to be known at t. Endowment and luck are also measured in capital-efficiency units. Accordingly, total child's wealth at t+1 is

$$I_{t+1} = w_{t+1}y_t + w_{t+1}e_{t+1} + w_{t+1}u_{t+1}$$
(4)

and "total family income," St, is defined as

$$Z_t + Z_{t+1}/D + I_{t+1}/D = I_t + w_{t+1}e_{t+1}/D + w_{t+1}u_{t+1}/D = S_t$$
(5)

The magnitude St in eq. (5) is the sum of the family's economic resources, its total wealth.

The parents decide on the allocation of the family wealth between present consumption, savings for their own future consumption, and their child's future capital. Optimal allocation maximizes utility in (1). Using (5), write the Lagrangian

$$H = U(Z_t, Z_{t+1}, I_{t+1}) - \lambda(S_t - Z_t - Z_{t+1}/D - I_{t+1}/D)$$
(6)

From the first order conditions one gets

$$\frac{\partial U}{\partial Z_{t}} = D \frac{\partial U}{\partial Z_{t+1}} = D \frac{\partial U}{\partial I_{t+1}}$$
(7)

Eq. (7) reflects the fact that, by the model, a dollar saved from consumption at t adds D dollars to consumption at t+1.

Rewriting eq. (7),

$$\frac{\partial U}{\partial Z_{t+1}} = \frac{\partial U}{\partial I_{t+1}}$$
(8)

$$\frac{\partial U}{\partial Z_{t}} \frac{\partial u}{\partial Z_{t+1}} = \frac{\partial U}{\partial Z_{t}} \frac{\partial U}{\partial I_{t+1}} = D$$
(9)

The meaning of eqs. (8) and (9) is that at the margin a dollar added to the parents' own future consumption is equal in utility to a dollar added to the future capital of their child. The generational discount factor is, in equilibrium, the marginal rate of transformation between present consumption and future outlays--whether in the form of parents' consumption or in the form of the child's capital.

Some of Becker's Findings

It will be useful to summarize at this point some of the findings of the <u>Treatise</u> which can be easily comprehended in light of the preceding analysis.

In a family with several children, equilibrium allocation of first period wealth will maintain equality of the discounted marginal utility of the income of all children; this marginal utility will also be equal to the present value of the marginal income of the parent's future consumption. Consequently, if parents are neutral in the sense that their marginal utility is the same function for all children, and the children are identical--all having the same endowment and luck--wealth will be allocated equally between the children. If, on the other hand, one child has comparatively higher endowment or luck (known, by assumption, at t), the parents will compensate his or her siblings with higher quantities of tangible capital. This is a manifestation of both the common pool of resources and parents' neutrality.

Because of its specificity, human capital (health, nutrition, schooling) is treated separately in the analysis. Assuming that the marginal productivity of small values of human capital is higher than the market rate of interest and that this rate is decreasing, families will invest "first" in the human capital of the offsprings and only "later" in tangible assets. Thus poor families may invest only in human capital and rich families will "satiate" their children with human capital and add on top of it tangible sources of income. As a result, earnings--returns to human capital--will be correlated with parents' income in poor families but not in rich ones.

Children differ in their ability to utilize human capital; therefore investment in the more able children yields higher returns than in the less fortunate offsprings. In this situation, the allocation of the parents' wealth is affected by two conflicting forces: productivity of investment and equality among the children. Depending on the relative importance of each of these forces, as expressed in the utility function, the family will invest comparatively more or less in the higher ability children.

Old Age Assistance

We are turning now to issues associated directly with farm transfer. It is common that the child who receives the farm also helps the parents in their old age, presumably from the farm proceeds. Does

this imply a special burden on the succeeding child? Within the assumptions of a single utility function, complete certainty, and identical rates of return, the answer to this question is negative. Differential returns may modify the conclusions as will be shown in the next section. Another factor which may modify the conclusion is uncertainty; it will be taken up later in the analysis.

A simple way to formulate old age assistance is to assume that the parents will receive in period t+1 a constant share, γ (0< γ <1), of the wealth of child 1, not necessarily the oldest child. More often than not, this will be the child continuing on the farm. With this assumption, the parents' utility function for a family with two children becomes

$$U = U(Z_t, Z_{t+1} + \gamma I_{t+1}^1, (1-\gamma) I_{t+1}^1, I_{t+1}^2)$$
(10)

A superscript indicates child 1 or 2 and Z_{t+1} is parents' consumption due to saving in their own "account" in period t. Total family income is modified only slightly

$$S_{t} = Z_{t} + Z_{t+1}/D + I_{t+1}^{1}/D + I_{t+1}^{2}/D$$
(11)

Two channels of saving will now support second period consumption of the parents: direct saving in their own account and indirectly through investing in child 1's capital. Maximizing U in (10) subject to (11), the parents determine how much they invest in each of their children's future wealth. By the first order conditions, the marginal utilities of the three arguments in (10) which are indexed t+1 will be identical; the amount of capital allocated to each child will however not be the same. To demonstrate in detail, it will be useful to introduce the following symbols

$$U_{1} = \frac{\partial u}{\partial Z_{t}}$$

$$U_{2} = \frac{\partial U}{\partial (Z_{t+1} + \gamma I_{t+1}^{1})}$$

$$U_{3} = \frac{\partial u}{\partial [(1-\gamma)I_{t+1}^{1}]}$$

$$U_{4} = \frac{\partial U}{\partial I_{t+1}^{2}}$$
(12)

In these symbols, the first order conditions yield

$$U_1 = DU_2 = DU_3 = DU_4$$
(13)

Equation (13) is too general for direct inference but, following Becker in adopting simplifying assumptions, we can gain further insight into the wealth allocation process. To this end, visualize the present and future wealth allocations, the magnitudes Z_t , $(Z_{t+1} + \gamma I^1_{t+1})/D$, $(1-\gamma)I^1_{t+1}/D$, I^2_{t+1}/D , as quantities demanded by the parents and assume homotheticity of the utility function. These assumptions imply constant expenditure shares. Let the shares be α_i (i=1, 2, 3, 4) such that

$$\Sigma \alpha_i = 1 \tag{14}$$

Parents' neutrality implies $\alpha_3 = \alpha_4 = \alpha$, there are no restrictions on α_1 and α_2 . Then

$$Z_{t} = \alpha_{1}S_{t}$$

$$(Z_{t+1} + \gamma I_{t+1}^{1})/D = \alpha_{2}S_{t}$$

$$(1-\gamma)I_{t+1}^{1}/D = \alpha S_{t}$$

$$I_{t+1}^{2}/D = \alpha S_{t}$$
(15)

The size of γ , the parents-child sharing parameter, does not affect the right-hand sides of (15), it only affects the distribution of wealth between the allocation to child 1's capital and to parents' saving. The values of these parameters are calculated from (15) as

$$\frac{I_{t+1}^{1}}{D} = \frac{\alpha}{1 - \gamma} S_{t}$$

$$\frac{Z_{t+1}}{D} = S_{t} (\alpha_{2} - \alpha \frac{\gamma}{1 - \gamma})$$
(16)

Old age assistance increases the amount of capital allocated to the succeeding child, but it reduces only the share of the parents' own saving. Each child receives the same net wealth

$$(1-\gamma)I_{t+1}^{1} = I_{t+1}^{2} = \alpha DS_{t}$$
(17)

Depending on the size of the parameters in the parenthesis in the second line of (16), Z_{t+1} may be zero or even negative; that is, the parents may rely for their future consumption solely on the farm

income, or they may even go into debt to finance family outlays in the first period. We shall further discuss debt in later sections.

Different Rates of Return

This section introduces the realistic possibility that returns are not identical. We continue to assume constant marginal (and average) rates of return, decreasing rates are taken up in the next section. Let the superscripts z, 1, 2 index the interest coefficients. Total family income in (11) is modified,

$$S_{t} = Z_{t} + Z_{t+1}/D^{z} + I_{t+1}^{1}/D^{1} + I_{t+1}^{2}/D^{2}$$
(18)

and the first order conditions yield

$$U_1 = D^z U_2 = D^1 [\gamma U_2 + (1 - \gamma) U_3] = D^2 U_4$$
(19)

With identical D values, equation (19) is reduced to (13).

The property of constant expenditure shares of homothetic utility functions is invariant to the values of the "prices," the D parameters. Accordingly, eq. (15) is now modified to read

$$\frac{(1-\gamma)I_{t+1}^{1}}{D^{1}} = \alpha S_{t}$$

$$\frac{I_{t+1}^{2}}{D^{2}} = \alpha S_{t}$$

$$Z_{t} = \alpha_{3}S_{t}$$

$$\frac{Z_{t+1}}{D^{2}} + \frac{\gamma I_{t+1}^{1}}{D^{1}} = \alpha_{4}S_{t}$$
(20)

Equation (16) is not changed, but (17) is modified as can be seen by comparing the first two lines in (20) to the corresponding expressions in (15). With identical rates of return the parents allocate equal shares of their wealth to each child and the children receive identical future wealth values. When rates of return are not identical, parents still parcel out identical shares of their wealth, but future incomes are not the same. This is analogous to demand for commodities: Consumers with homothetic utility functions allocate constant shares of their income to each commodity but the quantities they purchase vary with prices.

We have now demonstrated the mechanism of the family's wealth allocation when, as can be expected in general, returns in farming are not the same as returns in other pursuits. It is perhaps needless to add that the particular conclusions reached in the last paragraph are due to the specific assumption of homotheticity the way we defined it here. Parents may stick to equal future (planned) income of their offsprings and then--if rates of return differ--wealth allocation will not be in identical shares. (Becker discusses non-identical rates of return in Chapter 6 in the section titled "Compensation and Reinforcement of Differences among Children".)

Farm Succession and Wealth Allocation

Typically, farm assets and human capital are subject to decreasing returns. To incorporate this possibility in the analysis, consider a family with two children, one's comparative advantage is in farming and the other's in utilizing human capital in the form of schooling. The comparative advantage is in the generation of income, in the following sense: total family income, S_t , is highest when the advantages are realized and family capital is allocated optimally. The choice of the succeeding child, if there is more than one child, may be affected by timing considerations. In this chapter we disregard these issues and assume that timing of farm transfer is unimportant.

Marginal returns on investment in the farm and in human capital are decreasing; they are not the constant market rate D. Let upper indices f and h mark "farm" and "human;" D^{f} and D^{h} are average rates of return which are decreasing functions of the investment in each alternative

$$D^{f} = D^{f}(y_{t}^{f})$$

$$D^{h} = D^{h}(y_{t}^{h})$$
(21)

A rich family will assign to each child both the kind of capital which fits the child's comparative advantage and other non-human capital. The children will then have equal future wealth values, $I_{t+1}^{f} = I_{t+1}^{h}$. In a poor family, wealth will be given, at the margin, only in the form of farm assets and human capital. To simplify, assume that standard schooling (high school, perhaps) is given to each child and is included in the first period consumption variable Z_t . From this basis on, child f is provided by investment in the farm and child h--in schooling. We do not write explicitly now the endowment and market luck parameter and also assume that the cost of capital is $\pi_t = 1$. Then

$$I_{t+1}^{h} = y_{t}^{h} D^{h}$$

$$I_{t+1}^{h} = y_{t}^{h} D^{h}$$
(22)

For a poor family, the Lagrangian is

$$H = U(Z_t, Z_{t+1}, I^t, I^h)$$

- $\lambda (I_t - Z_t - Y_t - y_t^f - y_t^h)$ (23)

Maximization yields

$$\frac{\partial U}{\partial Z_{t}} = \frac{\partial U}{\partial Z_{t+1}} D$$

$$= \frac{\partial U}{\partial I^{f}} [D^{f} + y_{t}^{f} \frac{\partial D^{f}}{\partial y_{t}^{f}}]$$

$$= \frac{\partial U}{\partial I^{h}} [D^{h} + y_{t}^{h} \frac{\partial D^{h}}{\partial y_{t}^{h}}]$$
(24)

In (24) the derivatives in the square brackets are negative and the marginal rates of return in the farm and in schooling are lower than their averages. As indicated, this is the situation in a poor family. In a rich family children are provided with tangible "market" capital in addition to the wealth they receive in the farm and in schooling, then the marginal utilities in the last two lines of (24) are multiplied by the market rate D, and the farm capital and human capital are given to each child until for him or for her the marginal rate of return of the specific capital is equal to D.

In a rich family, if parents are neutral toward their children, total wealth of the two children is the same. In a poor family, maintaining the conditions of (24) may mean unequal distribution of future wealth. In Figure 5.1, children of a rich family get $I_r^{f} = I_r^{h}$, and children of a poor family have I_p^{f} and I_p^{h} , not necessarily the same. This is another demonstration of the potential conflict between equality and efficiency.

Farm and Debt

One advantage of owning a farm is that it can support debt. In our framework, debt is borrowed in period t and repaid in t+1. If the rate of interest on debt were lower than the rate of return on non-human assets, families would have borrowed indefinitely. This is generally not the case. So we let $r^b > r^n$, where b stands for "borrowing" and n indexes the market, non-human rate of return. With the above inequality, only poor families borrow and they borrow up to the point where the marginal rates of return on both human capital and farm assets are equal to the rate of interest.



Figure 5.1. Wealth in poor and rich families.

Borrowed money is fungible. There is no sense in asking, in the present framework, whether the money was borrowed to finance investment in farm assets or to finance schooling. It can be expected that the repayment of the debt in period t+1 will be the responsibility of one of the "accounts," either the parents will serve the debt or any of the children. The assignment of debt servicing will not affect the amount of net wealth allocated to any of the three accounts. This conclusion will not change if parents and children share equally or otherwise in the service of the debt. The conclusion may be different if uncertainty is introduced. Then the party responsible for repaying the debt may eventually find itself with a higher burden than was originally envisaged.

Farm and Non-farm Assets

In contradiction to the spirit of the modified Becker analysis presented thus far, it is often observed that the succeeding child gets a disproportional share of the family wealth. We discuss now several such cases.

Unequal allocation may be due to a number of factors which will be taken up one at a time. In all we assume that the family capital, K, is divided between the children and each child's capital contributes, with labor, to income. The family has two children, one of them the successor. To simplify, assume that the family has already decided on the amount of wealth to be allocated to the children; the question analyzed is the distribution of this wealth between the two children. The symbols are also simplified and the subscript t+1 is eliminated.

Specificity of Farm Capital

Rich farmers diversify their wealth. As management is non-divisible, poorer families may hold all their capital in the form of farm assets. Consider a poor family whose assets are farm specific--their contribution to production is higher than the amount they can be sold for. This means that capital allocated out of the farm is more expensive than capital transferred in the farm. Let the total value of the farm capital of a poor family be K, then

$$K^{f} + \beta K^{n} = K \qquad \beta > 1 \tag{25}$$

where the superscripts f and n stand for farm and non-farm allocations, respectively. The parents' utility function is written as

$$U = U(I^{f}(K^{f}, L^{f}), I^{n}(K^{n}, L^{n}))$$
(26)

with I^f and Iⁿ income of succeeding and non-succeeding child and $L^f = L^n = 1$ the corresponding labor inputs of the children.

From the first order conditions of the maximization of eq. (26), subject to (25), one gets

$$U_{1\overline{\partial}K^{f}}^{\underline{\partial}I^{f}} = \frac{1}{\beta}U_{2\overline{\partial}K^{n}}^{\underline{\partial}I^{n}}$$
(27)



Figure 5.2. Farm-specific capital (Eq. (5.27)).

Assuming, for concreteness, neutral parents (with identical marginal utility functions) and identical production functions for the two children, the succeeding child will get a larger share of the family wealth (Figure 5.2).

Overlapping Generations

Often the parents and the succeeding child expect to work the farm together for a long period, perhaps 10 or 20 years. If subjective discount rates are high, the periods further beyond are disregarded. The parents may then allocate a larger share of the family wealth to the farm which is expected to support two families. This situation can be expressed in the following way: The utility function is

$$U = U(I^{f}(K^{f}, L^{f}), I^{n}(K^{n}, L^{n}))$$

$$K^{f} + K^{n} = K$$

$$L^{f} = 2; \quad L^{n} = 1$$
(28)

Remember that two families now draw on the farm income. Constrained maximization yields,

$$\frac{\partial I^{f}(K^{f},2)}{\partial K^{f}} = \frac{\partial I^{n}(K^{n},1)}{\partial K^{n}}$$
(29)

Again, with identical production functions, the marginal product function for capital on the farm is higher (with two laborers) than that for the non-farm business and the farm will be allocated a larger share of the family assets.

Non-productive Value of Land

Farm land may have uses in non-agricultural activities. The succeeding child receives the land as a source of income in farming, sometimes against a promise to support the parents in their old age. In some cases it is even specified explicitly in the contract of the farm transfer that the succeeding child receives the farm to cultivate it and cannot sell its land for a given period of time. Formally,

$$U = U(I^{f}(K^{f}, L^{f}) + \alpha K^{f}, I^{n}(K^{n}, L^{n}))$$

$$(1 + \alpha)K^{f} + K^{n} = K$$

$$\alpha > 0$$
(30)

The coefficient α represents the additional non-agricultural value of the land; it is a decreasing function of the expected delay in realization.

From the first order conditions, one gets

$$\frac{U_1}{1+\alpha}\left(\frac{\partial I^f}{\partial K^f} + \alpha\right) = U_2 \frac{\partial I^n}{\partial K^n}$$
(31)

The allocation of the family wealth between the children will depend on the relative magnitudes of the functions and variable in eq. (31).

Reimbursement

Egalitarian distribution of the family wealth can be achieved with the succeeding child reimbursing the other child in cash. Lump sum payment may be too costly or even impossible--depending on the conditions in the financial markets. Instalment payment are like a loan the succeeding child receives from the other siblings to purchase their share in the farm. It raises questions of uncertainty for both sides. It also creates conflicts which are solved by bargaining or power struggle, possibilities we shall discuss in other chapters.

6. Overlapping Families and the Choice of the Successor Child

The decision on the succeeding child is one of the toughest a farm family has to make. We consider here one aspect of the choice problem; namely, the question of the appropriate time to expand the farm "population" to include two families. The analysis is conducted in the framework of a single utility function, viewing it from the point of view of the family as one unit. Potential conflicts will be taken up in later chapters.

As a rule, the succeeding child settles on the farm and establishes a family; farm labor supply increases and, at the same time, the number of people who draw their consumption from the income of the farm also rises. By postponing the transfer of the farm to a younger child, the parents may reduce the period in which two overlapping family generations live together on the same plot. This is one issue that is introduced here into the succession considerations.

The other major issue is life cycle productivity of the parents. Generally, it rises up to a certain age, flattens, and then declines. Early succession may mean that the parents either give up farming when they are still in prime age, or that a new family joins the farm when the labor supply of the parents is still high and the additional contribution of the young couple may be relatively small. Optimal timing of farm transfer, taking into account the income cycle of the parents, is developed by

Kimhi (1995c) in Chapter 11. Kimhi's analysis abstracts from the existence of several children and is conducted as if there was only one child who may take over the farm at an optimal date.

We assume that parents are not restricted in their choice, neither by law nor by custom; any child can be made to succeed. To concentrate on the main issue of overlapping generations, the analysis presented here disregards comparative advantage considerations and is conducted under the assumption that the children are of identical abilities.

Family transition

We demonstrate transition alternatives for a family with three children. Time in the transition period is divided into three sub-periods, each corresponding to the time at which one child, at its turn, may join the parents on the farm. For concreteness, one may think of three-year sub-periods, then the analysis covers nine years. Table 6.1 presents three modes of transition. In the table, F_i stands for the succeeding child i with i=1 for the first child and i=3 for the third. T_i symbolizes a child leaving the farm; P marks the parents. Thus in Part A of Table 6.1, the successor is the first child; it is the second child in Part B, and the third in C.

	Sub-period 1	Sub-period 2	Sub-period 3	Total
A. First child				
F_1	2	2	2	
T_2	1	0	0	
T ₃	1	1	0	
Р	2	2	2	
Family members	6	5	4	15
Laborers	4	4	4	12
B. Second Child				
T_1	0	0	0	
F ₂	1	2	2	
T ₃	1	1	0	
Р	2	2	2	
Family members	4	5	4	13
Laborers	2	4	4	10
C. Third child				
T_1	0	0	0	
T_2	1	0	0	
F ₃	1	1	2	
Р	2	2	2	
Family members	4	3	4	11
Laborers	2	2	4	8

Table 6.1. Family Transition with Alternative Succeeding Children

The table is constructed for two families of two adults, the parents or the succeeding couple. It is also assumed in the table that children, so long as they are young, live with their parents on the farm, share in consumption but do not contribute to farm labor.

To see how the table is read, consider Part B. The first child leaves the farm at the beginning of the first sub-period and he or she will not be there for any of the three sub-periods. This is indicated by 0 for each of the three sub-period columns in the T_1 row. The succeeding son or daughter will live with the parents as youngster in sub-period 1; the number 2 for the two other sub-periods is due to the assumption that a family of two is established once the second child joins the parents on the farm. The third child is still young in sub-period 1 and 2 and leaves the farm at the beginning of the third sub-period; a 0 value indicates this eventuality.

The "total" column in the table indicates total person-period, both for the number of people (family members) and the number of laborers on the farm, under the assumption that adult family members contribute to farm labor and children do not.

The table indicates a reduction of the total numbers of both laborers and person on farm, from Part A to Part C: with the move of the choice of the succeeding child from the first to the second and the third.

The model

The analytical model covers only the transition period, on the simplifying assumption that the choice of successor does not alter the economics of the family beyond that period. Also for simplicity, discounting is disregarded (zero interest rate is assumed). The choice of a particular child is the same as the choice of a transition alternative A, B, or C, in Table 6.1. An alternative is identified in the model by the total number of persons on the farm, which is taken as continuous.

As indicated earlier, the trade-off is between number of consuming family members and labor supply. On the one hand, the more people on the farm, the less there is for each of them. On the other hand, the more laborers, the higher is farm output. Let n stand for the number of family members, young and adult, on the farm. Taking the transition time as a single period, n is the total person-period, as in Table 6.1. The number of laborers is $n-\Delta$ ($\Delta=3$ in Table 6.1) and the income generating function is I($n-\Delta$). The symbol q stands for the amount of personally (privately) consumed goods and Z for the household (public) goods. The price of a unit of the personal good is p and the price of the household good is π . Family welfare is a function of the household good and the identical amount of private consumption q

$$u = u(q, Z) \tag{1}$$

The budget constraint is

$$pqn + \pi Z = I(n-\Delta) \tag{2}$$

Maximizing (1) with respect to n, Z, and q, subject to (2), yields the following first order conditions

and

$$n\left[\frac{\partial u}{\partial Z}\frac{\partial u}{\partial q}\right] = \frac{\pi}{p} \qquad (4)$$

$$I_{q} = \int_{\frac{\partial T}{\partial n}} \int$$

 $pq = \frac{dI}{dn}$



Figure 6.2. Equation (6.4).

(3)

The product pq is the "wage," or cost, of an additional laborer, and by (3), in equilibrium it equals the marginal product of labor on the farm (Figure 6.1). Equation (4) establishes the nature of good Z as the public good of the farm household. The expression on the left is the slope of the social indifference curve in Figure 6.2, the right-hand-side is the slope of the budget constraint.

Discussion

Equation (3) determines the number n and, in this way, the succeeding child. Eq. (4) determines the composition of consumption: private and household goods.

The variable n stands in equations (1) to (4) for the total number of family members, but it also represents the stage in the income cycle of the parents: a large value of n indicates early succession and an early stage in the income cycle. This fact is reflected in the income generating function $I(n-\Delta)$ and in its derivative in (3). For an internal solution, the right-hand-side derivative in (3) is decreasing; that is, going backward in succession, from the third to the second child and from

the second to the first, adds to the farm income, but at a decreasing rate. The decreasing marginal income is partly due to the movement backward in the income cycle of the parents. When the parents are young, they can perform well all farm chores and additional labor input does not add much to income; with older parents, the successors' labor is more effective (In an alternative formulation capturing productivity change, effective labor supply of the parents would change with age).

A comparative static analysis (not reported in detail) shows that an improvement in productivity (an increase in dI/dn) increases the equilibrium value of n and reduces the value of q. Productivity contributes to earlier succession though, by having this effect, it reduces per-capita consumption of the private good on the farm.

Thus far, the analysis has been carried under the assumption that the children are of identical abilities. If some are comparatively better than others in farming, the choice of the succeeding child may be modified (see chapter 10). However, the comparative advantage must be large enough to "over-rule" the timing consideration introduced in this discussion.

7. Sharing Farm Work

In the first stage of the family life, when the children are still young, the parents are the decision makers and the behavior of the family can be represented by a single (the parents') utility function. Once the children are grown and a succeeding son was chosen, the family members are adults--each with his or her own utility function. In this chapter we analyze the contractual relations between the father and the succeeding son in the period in which they work the farm together. Several types of models will be considered in search for an appropriate contract.

The Sharecropping Analogue

By this model, in period one the father and the succeeding son work together on the farm and share its income. In period two, the farm is worked only by the son and the father receives a fixed annuity. Capital invested by the son at the beginning of period one contributes to production in both periods, the investment is financed by a loan repaid in the second period.

By construction, the father sets the parameters of the contract and the son, once accepting the offer and the responsibilities that go with it, maximizes his income given the offered parameters. No altruism or gift is assumed apart from the fact that the son receives eventually his father's farm. In this way the father-son relationship takes the form of the commercialized landlord-tenant relation. Consequently, our findings also follow those reported in the sharecropping literature; they will be interpreted here in the terms of the farm transfer process.

The Model

The son divides his labor in both periods between farm work, L't, and non-farm work, L"t,

$$L_{t}^{'} + L_{t}^{"} = L_{t}$$
 t=1,2 (1)

The father's labor input on the farm in period one is L (without a time index).

Farm production is

$$Q_1 = F(1)(L'_1 + L, K, T)$$
 period one (2)
 $Q_2 = F(2)(L'_2, K, T)$ period two (3)

where K is capital assets installed at the beginning of the first period and T stands for land and other predetermined factors of production. The functions F(t)() are well behaved; they may be identical, the argument t is introduced for convenience--to distinguish between the production processes in the periods.

Let w be wage rate, the alternative income the son can earn in non-farm work. The share of the son in period one's farm product is α ($0 \le \alpha \le 1$); for the time being, α is given exogenously. Accordingly, income in period one is

$$Y_1 = \alpha Q_1 + w L_1'' \qquad \text{son (4)}$$

$$y_1 = (1 - \alpha)Q_1 \qquad \text{father} \quad (5)$$

Income in period two is

$$Y_2 = Q_2 - DK - \beta + wL_2''$$
 son (6)

$$y_2 = \beta$$
 father (7)

where D = (1 + interest rate) and β is the fixed income the father receives in the second period. The present value of the two period streams of income for the son and the father are

$$Z = Y_1 + D^{-1} Y_2$$
 (8)

$$z = y_1 + D^{-1}y_2$$
 (9)

The son maximizes Z given α and β ; the maximization is with respect to L'1, L'2, and K. First order conditions are

$$\alpha \frac{\partial F(1)}{\partial L_1} - w = 0 \tag{10}$$

$$\frac{\partial F(2)}{\partial L_2} - w = 0 \tag{11}$$

$$\alpha \frac{\partial F(1)}{\partial K} + D^{-1} \frac{\partial F(2)}{\partial K} - 1 = 0$$
(12)

Conditions (10) and (12) are the familiar "Marshallian inefficiency" conditions in sharecropping (for $\alpha < 1$): the tiller who receives as his share only part of the farm product, under-utilizes the factors of production he pays for, explicitly or implicitly. In our case, the son works too little on the farm and too much off it, and similarly his investment is suboptimal.

Alpha Contract

We saw in the previous section that the son works too little on the farm if he gets only part of its product. The question may be raised, Can the father amend this shortcoming of the contract by choosing an appropriate magnitude for α ? The analysis in this section will demonstrate that complete elimination of the shortcoming is impossible and will quantify the gap between the father and the son.

The father chooses the value of α , the annuity β is predetermined. In choosing α the father maximizes his income, given the son's reaction to the value of the parameter. In addition, for the son to work on the farm in the first period, his share of the farm product must be at least as high as his alternative income opportunity. Hence the father's maximization is done under the constraint

$$\alpha Q_1 + w L_1'' \ge w L_1 \tag{13}$$

Assuming an internal solution and that the constraint is binding, eq. (13) can be expressed as

$$\alpha Q_1 = w L_1 \tag{14}$$

and the father maximizes H in the following equation

$$H = Q_1 - wL_1' + D^{-1}\beta$$
 (15)

H is maximized with respect to α :

$$\frac{\partial H}{\partial \alpha} = \frac{\partial L_1}{\partial \alpha} \left[\frac{\partial F(1)}{\partial L_1} - w \right] + \frac{\partial F(1) \partial K}{\partial K \partial \alpha} = 0$$
(16)

(17)

In choosing α , the father attempts to direct his son's labor allocation. Keeping this aim in mind, the first order condition in (16) can be examined from two points of view. To exhibit the father's "plan" for the son's work, eq. (16) can be rewritten as



Figure 7.1. Son' s labor on the farm

The amount of the son's labor consistent with (17) is marked as L(F) in Figure 1: the father attempts to direct the son to work on the farm more than socially optimal. On the other hand, by eq. (10), in deciding on his labor input on the farm, the son equates labor's marginal product with w/ α . This point is marked L(S) in the figure. The discrepancy between L(S) and L(F) is, in the current formulation, a measure of the magnitude of the gap between the father's position and that of the son. The conflicting positions and the impossibility of their reconciliation can also be seen if the son's first order condition (10) is inserted into (16)

$$\frac{\partial H}{\partial \alpha} = \frac{\partial L_1}{\partial \alpha} w [\frac{1 - \alpha}{\alpha}] + \frac{\partial F(1) \partial K}{\partial K \partial \alpha} \neq 0$$
(18)

In general, no value of α can set eq. (18) to equal. A solution to the father-son contractual conflict does not exist under the current formulation. As we show below, the contractual conflict can be resolved if the father sets both α and β , but the solution is then a corner solution.

Alpha Beta Contract

Now the constraint (assumed binding) is that the present value of the son's income on the farm is equal to his alternative earning. Accordingly, the father maximizes H in the following with respect to α and β

$$H = (1 - \alpha)Q_1 + D^{-1}\beta + \lambda[\alpha Q_1 - K - D^{-1}\beta + D^{-1}Q_2 - wL_1' - D^{-1}wL_2']$$
(19)

The first order conditions are

$$\frac{\partial H}{\partial \alpha} = -Q_1 + (1-\alpha) \left[\frac{\partial F(1)\partial L_1'}{\partial L_1'} + \frac{\partial F(1)\partial K}{\partial K \partial \alpha} \right] + \lambda \left[Q_1 + \alpha \frac{\partial F(1)\partial L_1'}{\partial L_1'} + \alpha \frac{\partial F(1)\partial K}{\partial K \partial \alpha} - \frac{\partial F(1)\partial K}{\partial K \partial \alpha} \right]$$
(20)
$$- \frac{\partial K}{\partial \alpha} + D^{-1} \frac{\partial F(2)\partial K}{\partial K \partial \alpha} - w \frac{\partial L_1'}{\partial \alpha} \right] = 0$$
(21)

Inserting $\lambda = 1$ from eq. (21) and the son's first order conditions (10) and (12) into (20) yields

$$\frac{\partial H}{\partial \alpha} = (1 - \alpha) \left[\frac{\partial F(1) \partial L_{1}}{\partial L_{1}} + \frac{\partial F(1) \partial K}{\partial K \partial \alpha} \right] = 0$$
(22)

The equality in (22) is satisfied at $\alpha = 1$. This is a corner solution the meaning of which is that the father maximizes his income (its present value) if he offers the son all the farm product in period one and gets the maximum value of β in two:

$$\beta = DQ_1 - K + Q_2 - DwL_1 - wL_2$$
 (23)

For the son, the payment of β in period two is a lump-sum payment and its value does not affect his labor allocation in either period. Receiving the full value of Q₁ in period one and paying a given β in period two, the son equates the marginal product of farm labor in both periods to the off-farm wage and allocates his labor optimally.

The contract which we have now formulated requires the father to work in period one full time on the farm while his returns are due only in period two. He receives no income in period one and has then, by the solution, to draw on his savings or to take a loan to finance his consumption in this period. Such contracts will seldom, if ever, be observed.

Risk Sharing

Continuing with the analogy to the relations between the landlord and the tenant, we demonstrate now a simple case of risk-sharing. Assume that farm output in period one is subject to random disturbance, unknown when input allocation is determined. Accordingly, we write now

$$Q_1 = \Theta F(1)(L_1 + L, K, T)$$
(24)

where θ is the disturbance term with $E\theta = 1$. Other sources of uncertainty are disregarded at this point.

The father and the son strive to maximize expected utility. For simplicity, income is the only argument in the welfare functions. The son's utility is

$$U = U(\alpha Q_1 - K - D^{-1}\beta + D^{-1}Q_2 + w(L_1 - L_1') + D^{-1}w(L_2 - L_2'))$$
(25)

and his opportunity or "reserved" utility is

$$U = U(wL_1 + D^{-1}wL_2)$$
 (26)

The utility of the father is

$$v = v[(1 - \alpha)Q_1 + D^{-1}\beta]$$
(27)

Again for simplicity of exposition, we assume now that the father (not the son) decides on the amount of labor the son will contribute on the farm. Formally the father maximizes his expected utility with respect to L'_1 , α , and β , subject to the constraint that the son's utility equals its reserved value

$$H = Ev + \lambda(EU - U) \tag{28}$$

The first order conditions are

$$\frac{\partial H}{\partial L_{1}'} = (1 - \alpha) E \nu' \Theta \frac{\partial F(1)}{\partial L_{1}'} + \lambda [\alpha E U' \Theta \frac{\partial F(1)}{\partial L_{1}'} - E U' w] = 0$$
⁽²⁹⁾

$$\frac{\partial H}{\partial \alpha} = -Ev' \Theta F(1)(1) + \lambda EU' \Theta F(1)(1) = 0$$
(30)

$$\frac{\partial H}{\partial \beta} = D^{-1} E v' + \lambda [-D^{-1} E U'] = 0$$
(31)

Combining eqs. (30) and (31), the risk-sharing equilibrium condition for α and β is

$$r = \frac{Ev'}{Ev'\Theta} = \frac{EU'}{EU'\Theta} = R$$
(32)

By eq. (32), the father's risk measure r is equal in equilibrium to the son's R. Figure 7.2 illustrates graphically the difference between the nominator and the denominator in the ratio which forms the measure of risk. In the figure, θ is assumed to take the values 0.5 and 1.5, each with probability 0.5. Decreasing risk (in the diagram to $\theta = 0.8$ and $\theta = 1.2$) reduces risk as defined in eq. (32).



Figure 7.2. Construction of risk measures in Eq. (7.32), $\alpha = 1$, $\beta = 0$.

For a constant marginal utility function of the son (U' = constant), in eq. (32) R = 1 and by the equation, r = 1: no risk is imposed on the father. This can be verified by examining (29). For U' = constant, the equation vanishes for $\alpha = 1$ and the solution returns to the corner case of the last section where the father's only income comes in the second period.

For U' not necessarily constant, the condition for the allocation of the son's labor is

$$\frac{\partial F(1)EU'\Theta}{\partial L_{1}^{'} EU'} = w$$
(33)

Marginal product of labor multiplied by the son's risk measure is equal to his off-farm wage rate.

Nash Bargaining Solution

By construction, the Alpha Contract above was to offer the son his reservation utility and the father, if he chooses to offer such a contract, receives the rest of the farm's income in period one. Thus the father receives all the "profits" of the deal. In practice the two parties may negotiate the terms of the contract and share the benefits of their cooperation. Continuing with the simplifications of the earlier parts of the paper, we identify utility with income. Write Z_0 and z_0 respectively for the son's and father's alternative income. Then by Nash, accepting a set of plausible assumptions, the value of α agreed upon by the father and the son as the solution to their bargaining will maximize the product

$$(Z - Z_0)(z - z_0)$$
(34)

It will be useful to start now with a couple of preliminaries. Differentiating Eq. (10) and rewriting, one gets

$$\frac{dL_{1}^{'}}{d\alpha} = -\frac{\frac{\partial F(1)}{\partial L_{1}^{'}}}{\alpha \frac{\partial^{2} F(1)}{\partial L_{1}^{2'}}} > 0$$
(35)

As α grows, the son contributes more labor on the farm. This expression will be useful in eq. (38) below.

The term z_0 stands for the income the father will have if no agreement is reached. Then the father will have to work the farm on his own in both periods. To distinguish this possibility from joint cultivation, we write the production functions for the two periods as

$$QF_t = F(t)(L, 0, T) \quad t = 1, 2$$
(36)

Note that in eq. (36), L' = 0 and K = 0, the son does not work on the farm and does not invest in its assets.

Now, spelling out eq. (34), the chosen α will maximize

$$(Y_{1} + D^{-1}Y_{2} - WL_{1}^{'} - D^{-1}WL_{2}^{'})(y_{1} - D^{-1}y_{2} - QF_{1} - D^{-1}QF_{2})$$

$$= [\alpha Q_{1} - WL_{1}^{'} + D^{-1}(Q_{2} - DK - \beta - WL_{2}^{'})]$$

$$[(1 - \alpha)Q_{1} - QF_{1} + D^{-1}(\beta - QF_{2})]$$
(37)

The first order condition for maximizing eq. (37) with respect to α is [making use of eqs. (10) and (12)]

$$Q_{1}[(1 - \alpha)Q_{1} - QF_{1} + D^{-1}(\beta - QF_{2})]$$

$$+ [\alpha Q_{1} - WL_{1}^{'} + D^{-1}(Q_{2} - DK - \beta - WL_{2}^{'})]$$

$$[-Q_{1} + (1 - \alpha)\frac{\partial F(1)dL_{1}^{'}}{\partial L_{1}^{'} d\alpha}] = 0$$
(38)

Empirical implementation will be postponed to a later study.

8. Transfer of Ownership

As already explained, the designation of a youngster as a "continuing son," and even the acceptance of that son or daughter to membership in the cooperative association of the moshav, does not constitute a formal and legal transfer of ownership of the farm. This transfer is a separate act to be registered with the Land Authority. So long as the transfer has not been formally registered, the farm belongs to the parents and they may bequeath it to whomever they choose. So long as the farm has not been formally transferred to the successors, the young family cannot be sure that the farm will eventually belong to it.

Uncertainty is not eliminated, however, by the legal transfer of the farm. Parents usually stay on the farm and live of its income as part of the explicit or implicit agreement accompanying the transfer. However, once the farm is officially in the hands of the succeeding family, the retiring parents face the risk of losing their initially agreed upon rights. The young couple may be either unwilling or unable to secure for the old parents the standard of living they have expected to enjoy. In this chapter we consider the transfer problem and decisions under uncertainty in two settings: (a) a two-period framework incorporating questions of productivity and investment; and (b) the timing of legal transfer which will be considered in a multi-period model. For brevity and simplicity, the discussion is conducted in father-son terms.

The Conditions of Transfer

It is reasonable to assume that once the farm is transferred to the son, he will readily invest more in its development than if his ownership is not secured. On the other hand, keeping the ownership is the father's defense against the possibility that the son will default on the agreement. Generally, an agreement is not reached if either party deems the no-agreement alternative superior to any other. In the circumstances of farm transfers, either the father or the son must be compensated if the farm is to stay in the possession of the other.

The Contract Game

The possibilities open to the father and the son can be formulated as a two-party game. We attempt to cover the essential components of the situation in a simplified framework depicted in the extensive form representation in Figure 8.1. Two periods are considered; for concreteness, imagine the game to commence when the father is 50 years old and each period to last 10 years. The "game" ends after 20 years; that is, the planning horizon is 20 years. At point 0 the father and the candidate to succeed him either reach an agreement (branch 1) or they do not (branch 2). An agreement is reached if any of the possibilities marked 3 and 4 is not dominated by the no-agreement alternative for both the father and the son.

The agreement can be either that the father continues to keep the farm for the first period (10 years) or that the farm transfer is executed immediately. If the father keeps the farm (3) he may, in period two, continue holding it or bequeath his wealth to someone else (5) or, alternatively, he may maintain the agreement and transfer the farm to the succeeding son (6). If the father keeps the farm (3), the probabilities of realizing the final outcomes 5 or 6 are q and 1-q, respectively. Similarly, if the son receives the farm, the probabilities are p and 1-p that the final outcomes on branches 7 and 8 will be realized. These probabilities may be subjective; as explained below, they will be considered here as objective.

We continue with the symbols introduced in the earlier chapters. The son receives a share α of the farm income in the first period. The father receives 1- α and β , respectively, as his share in the first period's income and an annuity in the second period. In addition, two modifications are introduced: a. In period two, the father receives a share, γ , of the farm income (as well as the annuity β); and b. To distinguish between ownership possibilities, capital invested in period one is now marked KFS if the father keeps the farm (branch 3) and KSS for branch 4. Similarly, farm output for these two cases is marked QFS and QSS. Consistently with the hypothesis that the son will invest more in the farm if his ownership is secured, it is assumed that KSS > KFS and QSS > QFS.



Figure 8.1. Father–son contract tree.

The normal form representation of the game is shown in Table 8.1. In the table, F2, F5, F6, and F4, are the father's strategies; the son's strategies are S2, S7, and S8. The digits in the strategy symbols indicate the numbering of the branches of the tree in Figure 8.1. For example, F5 is the strategy: keep the title to the farm for the first period and deprive the son of the farm in the second. A probability q is attached to this event.

	S2	S7 (p)	S8 (1-p)	
F2	a2, b2	a2, b2	a2, b2	
F5 (q)	a2, b2	a5, b5	a5, b5	
F6 (1-q)	a2, b2	a6, b6	a6, b6	
F4	a2, b2	a7, b7	a8, b8	

Table 8.1. Father-Son Contract Game Parametric Representation

Fi--father, Si--son.

The extensive form tree is depicted in Figure 8.1.

The strategies, Fi and Si, and the payoffs ai, bi are indexed in accordance with Figure 8.1.

Either party – the father or the son – may veto an agreement and, accordingly, the payoffs are identical (a2, b2) for all the entries in the F2 row and S2 column which stand for the no-agreement strategy. By construction, if not rejecting an agreement, the father and the son act at different nodes

of the tree. Thus the father may offer the son branch 3 or 4 and, if 3 is taken, the father will be the only player at the beginning of the second period. If branch 4 is taken, the son will be the sole player at the next node. The payoffs in the game are affected by the actions of the son only in row F4; only in this row the entry for column S7 differs from that for column S8.

In a regular game the parties choose strategies. If mixed strategies are chosen, each player sets the frequencies (or probabilities) at which the different pure strategies will be played. In the father son game, the probabilities are subjective and the parties may affect beliefs to some extent. But one may also think of exogenously given probabilities: there exists a certain probability that one of the parents will die before the farm was transferred and the surviving spouse will remarry and deprive the succeeding couple of their promised share. Similarly, there is a certain probability that the succeeding couple will separate and not be able to keep their promise to let the parent stay on the farm. Such possibilities form the objective probabilities attached to the branches of the tree in Figure 8.1, even if the father and the son have no prior intentions to default when the occasion arises. Assume that these objective probabilities are known to the parties; they may have deduced them from the history of other families in moshavim. Consequently, the control variables in the game they play are not mixtures of strategies but rather the rewards they offer.

We shall now consider as an example a game in which the father and the son agree on branch 4; that is, the son receives the farm in period one. The question we ask is, given the data of the game, what is the minimum payoff that the father has to receive in order to agree to branch 4? If the father actually gets that minimum, the son will collect the entire "rent" of the game. The father and the son negotiate on three parameters: α , β , and γ . As we shall see shortly, in the game of Figure 8.1 and Table 8.1 the values of the parameters are solved in a two equation system. Only two parameters can be determined; let β be given.

No agreement will be reached if branch 2 dominates all other possibilities (if either entry in a2, b2 dominates all other corresponding entries in Table 8.1). The game is presented for the case in which at least one agreement strategy dominates.

The father will agree to branch 4 if it is at least as good as his best alternative. The father's alternative is to offer branch 3. The son will take branch 3 if the following inequality is maintained

$$b5q + b6(1-q) \ge b2$$
 (1)

Mark the maximum value of the game for the father, when branch 3 is agreed upon, as MF

$$MF = a5q + a6(1-q) \tag{2}$$

Given eq. (2), the father may agree to branch 4 if

$$MF \le a7p + a8(1-p) \tag{3}$$

In an agreement in which the son gets the farm in the first period (branch 4) and the father receives the minimum needed to convince him to accept this arrangement, the inequalities in eqs. (1) and (2) become equalities. Now, equations (1)-(3) can be constructed as a two equation system and

solved for the two unknowns: the shares α and γ . We turn now to a numerical demonstration of this possibility.

An Example

Table A1 in the Appendix specifies income components of the father and the son under the different circumstances they may face as depicted in the tree diagram of Figure 8.1. The values chosen for the parameters in the numerical example are presented in Table A2. The construction of the actual rewards, the payoff of the game, is done in Table A3. The final magnitudes (such as 143.043 for a5) are already equilibrium values. These values are also the entries in the normal form representation of the game in Table 8.2.

Table 8.2. Father-Son Contract Game, The Normal Form

	S2	S7	S8	
F2	87, 114	87, 114	87, 114	
F5	87, 114	143, 42	143, 42	
F6	87, 114	122, 153	122, 153	
F4	87, 114	107, 223	133, 197	

Fi--father, Si--son.

The extensive form tree is depicted in Figure 8.1.

See the Appendix for the construction of the numerical values in the table.

The equilibrium values were obtained when the equations (1) - (3) were solved for the numerical values of the example as given in the Appendix tables. The solution was

$$\alpha = 0.1087, \quad \gamma = 0.0525$$

Note that in Table 8.2, no-agreement is dominated, both for the father and the son, by some alternative outcomes. It may be therefore in the interest of each of the parties to reach an agreement, as was assumed in the above. The value of the contract is MF for the father and MS for the son:

$$MF = 107p + 133(1-p) = 129$$
$$MS = 223p + 196(1-p) = 200$$

The solution and the worth of the contract will change with a modification of the parameters. For example, numerically calculated comparative statics values are shown in Table 8.3. By the table, an increase, for example, in p by one percentage point, causes an increase in the equilibrium value of α by 0.39 of a percentage point (as p is changed from 0.15 to 0.16, α is modified from -0.1087 to 0.1128). A rise in p is an increase in the risk the father faces should the son take the farm in the first period; an increase in q is a rise in the risk the son faces if the father keeps the farm. Both changes

modify the time distribution of the payoffs in the same direction: the son gets more in period one and the father gets more in period two.

Table 8.3. Elasticity Values (in Equilibrium)

	р	q	
α	0.39	1.49	
Y	0.57	0.64	

Timing of Legal Transfer

By the regulations of the moshav, actual and legal farm transfer can take place any time after a child was declared the "continuing son." In practice, it is common for parents and the succeeding couple to cultivate their farm together without a formal and explicit contract for many years. Ownership is formally transferred only after a substantial period of joint cultivation and sometimes only after the parents' death.

There can be several reasons for the delay in contract signing and farm transfer. Explicit discussion of division of assets and retirement provisions of the parents while they are still relatively young may entail psychological difficulties giving rise to intra-family tension and "transaction costs." Early contracting and farm transfer to an elder child may seal the fate of other offsprings while they are still too young to have their own opinion or to voice it effectively. The parents may wish to have a period of trial and adjustment at which they can examine life together--with the succeeding child now as an adult person and with the incoming and often unaccustomed spouse. The young couple may wish also to accumulate experience on the farm before they settle for good and before they commit themselves to support the parents in the future.

Farm transfer, being a long run contractual arrangement, is associated with uncertainties of many kinds. One of them looms large, by the Law of Inheritance the last will dominates any previous contract or promise. Consequently, the succeeding son, if the farm was not actually and officially transferred to his name, may find after many years of joint cultivation that the farm was not bequeathed to him. The father may effectively remove him from the farm even before his death by simply disclosing the contents of his will. As a safeguard against these possibilities the son may be interested in early transfer.

The parents also face uncertainties. With the years, they may become disappointed with the son or the spouse. This is a particular danger once the farm was legally transferred; then, if a serious disagreement arises, the young couple may fail to meet their contractual obligations or cause other hardships. So long as the parents keep legal ownership of the farm, they are in a better position to protect their rights in property and income.

One aspect of the succession process is management and decision making. In some cases, the father may step aside and let the son make all the technical and economic decisions right from the day the son joins him on the farm. In other cases, the father runs the farm until the moment of legal transfer. Sometimes, to avoid conflicts, the farm activities are divided; for example, the father may continue

with livestock and the son will get the crop and horticultural activities to run independently. One reason for potential divergences of opinion on the management of the farm may be different attitudes toward risk. The young son may turn to off-farm activity if an agricultural venture fails; the father faces a constrained set of alternatives.

We suggest in this part of the chapter a model of the determination of the date of actual farm transfer. The possibility of official contracting without transfer will not be considered at this stage. It is assumed in the analysis that the son has two mutually exclusive sources of income: he either works the farm, or is removed from it and takes an alternative occupation. Standard of living on and off the farm are the same; the only difference being that on the farm the son can accumulate more saving than off it. Accordingly, if removed, the son loses the assets accumulated up to the time of removal. This loss will be the only danger the son is facing in the chapter's model. With these simplifying assumptions, the son's interest is to execute the transfer sooner rather than later while the father tries to postpone the legal act. The analytical task is to identify the factors determining timing of farm transfer. We start with a simple dynamic programming model at which the son faces uncertain future and has to decide whether he stays on the farm or leaves. By the model, the son faces this question every year until he either leaves or receives the farm legally.

Modeling the Son

The model is actually a one-person game, with the son the only player; the father is treated as if he were a mechanical lottery which the son is playing. The game lasts for T years. The son's decision points are at the beginning of each year: t=0 is the beginning of year 1, t=1 is the beginning of year 2, and so forth. At the beginning of every year the son decides whether to stay on the farm or to leave it. Once he left he cannot return to the farm (see Figure 8.2). At the end of each year the father acts in one of three ways: 1. He transfers the farm legally to the son, with probability PG; 2. He disinherits the son, with probability PD; 3. He does neither, with probability PN (PG+PD+PN=1). The probabilities can be subjective magnitudes the son attaches to his father's actions or objective probabilities associated with the actions of the father is the same for all T years except for the very last. At t=T, the son either inherits the farm with probability PGT or is disinherited with probability PDT=1-PGT.

The game ends in one of four ways: 1. The son leaves; 2. The son receives the farm; 3. The son is disinherited; 4. The point of time t=T is reached. The first three possibilities can occur at any t < T.

Working on the farm, the son accumulates S dollars per year; if working off the farm he accumulates aS (0<a<1) per year. When leaving the farm of his own will at t<T, the son takes with him a share b of the wealth accumulated on the farm; that is, at t he takes btS ($0 \le b \le 1$). If disinherited at t \le T, he receives nothing. The son is risk averse and he maximizes the expected value of a concave utility function defined on his wealth.

Three points are worth noting here: 1. Once the son received legal ownership of the farm, he has wealth TS with probability one. 2. The son faces uncertainty when working on the farm; off the

farm he is, by the model, in a world of certainty. 3. The son cannot threaten to leave. He plays against a "machine" (which is indifferent to his threats) and he either stays or goes.

The son's decision problem can be solved in dynamic programming. For any t<T, the utility off the farm (the reservation utility) is

$$U_{t} = U(a(T-t)S + btS)$$
(4)

(5)

For any t<T-1, the utility of a maximizing son, if he is still on the farm at t (and not its owner yet), is written in the form of a recursion functional as



Father

D=disinherits the son N=does not transfer (and not D)

Nodes \odot =decision (of son) O = game of chance

Figure 8.2. Father-son tree/

In eq. (5), EMU stands for the maximum value of the right-hand side and EU* stands for the expected utility of staying on the farm for another year. For t=T-1

$$EMU_{T-1}^{*} = \max[PGT^{*}U(TS) + PDT^{*}U(0), U_{T-1}]$$

$$= \max[EU_{T-1}^{*}, U_{T-1}]$$

$$U_{T-1} = U(aS + b(T-1)S)$$
(6)

The second lines in (5) and (6) are calculated in the numerical illustration in Table 8.4 below.

In words, the programming starts at t=T. Then, if the son is still on his father's farm, he receives the farm with probability PGT or is disinherited. Anticipating this, at t=T-1, the son compares the expected value of staying on the farm for the last year against the alternative of leaving it at this point. At t=T-2, the son compares the utility of leaving to the expected utility of staying, taking into account his optimal decision if he stays for another year.

Following this logic, the dynamic programming procedure is run backwards from t=T to t=0. When offered to be a succeeding son, the young family compares the expected value of the offer against the alternative of leaving immediately. The value of the utility at t=0 is calculated taking into account the possibility of leaving at some t in the future, if the farm is not legally transferred by then.

A Numerical Illustration

The illustration is presented in Table 8.4. The parameters are the following:

- horizon T=20;
- saving on farm S=100 dollars per year;
- off farm saving a=0.30 (30 dollars per year);
- share of accumulated wealth taken when leaving b=0.45;
- at t<T, probability of legal transfer PG=0.3;
- probability of being disinherited PD=0.2;
- the complement of the last two PN=0.5;
- at t=T, probability of inheriting the farm PGT=0.95;
- probability of being disinherited PDT=0.05;
- utility $U=V^{0.5}$ where V is wealth.

Time is presented in Table 8.4 from t=0 to t=19; the last is the beginning of year 20. At the end of this year the son will find (if he stayed up to this time and did not become the legal owner) whether he inherits the farm (with probability 0.95) or he is disinherited. The maximum value of the farm accumulated wealth is 2000. If the son leaves at t=0, he will accumulate off the farm 600 (Col. D). The value of the wealth of a son who leaves at t is calculated in Col. E; it is the present value of the savings accumulated off the farm (under our assumption of zero discount rate) plus 45% of the wealth accumulated on the farm up to this point.

t	S=100*	a=0.3*	(T-t)as+	U-BAR	EU*
	TS	(T-t)as	0.45tS		
0	2000	600	600	24.49	26.83
1	2000	570	615	24.80	26.83
2	2000	540	630	25.10	26.83
3	2000	510	645	25.40	26.83
4	2000	480	660	25.69	26.84
5	2000	450	675	25.98	26.84
6	2000	420	690	26.27	26.85
7	2000	390	705	26.55	26.85
8	2000	360	720	26.83	26.90
9	2000	330	735	27.11	26.97
10	2000	300	750	27.39	27.11
11	2000	270	765	27.66	27.25
12	2000	240	780	27.93	27.38
13	2000	210	795	28.20	27.51
14	2000	180	810	28.46	27.65
15	2000	150	825	28.72	27.81
16	2000	120	840	28.98	28.79
17	2000	90	855	29.24	30.75
18	2000	60	870	29.50	34.66
19	2000	30	885	29.75	42.49
at	T (0.95*S	SORT(2000)))		42.49

Table 8.4. Numerical illustration

EU at T (0.95*SQRT(2000))

Column F reports the reservation utility value for a son who leaves at t to works off the farm. Col. G is the expected utility of staying on the farm, the EU* values in eqs. (5) and (6). It was calculated backwards, first by (6) for T-1 and then by (5).

Comparing Cols. F and G, one finds that the son will accept the offer to become a succeeding son, since at t=0 the utility of this possibility (col. G) is higher than the alternative. But at t=9 the son will leave the farm if he does not receive its legal ownership by then (and was not disinherited). The present values, for each t, of staying or leaving the farm are also depicted in Figure 8.3.

The numerical example demonstrates--though not strongly--an "immobility trap." If the young family does not leave at t=9 and is "lazy," it may reach a time in which the optimal policy will again be to stay on the farm. In Table 8.1 this switching moment is at t=17. Quite far away, to reach it the young family has to be "immobile" for 8 years. Yet this possibility exemplifies the statement often made that "it is time to leave now before it becomes too late."



Figure 8.3. Dynamic programming

Appendix: Father-Son Contract

	Period One	Period Two
Agreement not Reached		
Father	y2.1 = QFF1	y2.2 = QFF2
Son	Y2.1 = wL1	Y2.2 = wL2
Agreement Reached		
Father	$y3 = (1-\alpha)(QFS1-KFS)$	y5 = QFF2
		$y6 = \beta + \gamma QFS2$
	$y4 = (1-\alpha)(QSS1-KSS)$	y7 = 0
		$y8 = \beta + \gamma QSS2$
Son	$Y3 = \alpha(QFS1-KFS) + wL1"$	Y5 = 0
		$Y6 = (1-\gamma)QFS2 - \beta + wL2''$
	$Y4 = \alpha(QSS1-KSS) + wL1"$	Y7 = QSS2 - DKSS + wL2''
		$Y8 = (1-\gamma)QSS2 - \beta + wL2''$

QFF1 farm production in period one with father the only worker; QFS1 the same when father and son work the farm; QSS2 only the son works the farm in period two. KFS is investment on farm when father keeps the title to the farm. Other variable are defined in a similar fashion.

Farm production	QFF1 = 60; QFF2 = 30; QSF2 = 50; QFS1 = 120; QSS1 = 140; QFS2 = 130; QSS2 = 180
Off farm labor	wL1 = wL2 = 60; WL1" = 30; WL2" = 20
Investment	KFS = 10; KSS = 20.
Discount factor	D = 1.1; 1/D = 0.9
Distribution parameters	$\beta = 20$; α , γ to be determined endogenously
Probabilities	p = 0.15; q = 0.35

Table A2. Income Alternatives, Numerical Magnitudes

Table A3: Payoffs

Father	
	a2 = y2.1 + y2.2/D = QFF1 + QFF2/D = 60 + 0.9x30 = 87
	$a5 = y3 + y5/D = (1-\alpha)(QFS1-KFS) + QSF2/D = (1-\alpha)(120-10) + 0.9x50$
	$a6 = y3 + y6/D = (1-\alpha)(QFS1-KFS) + [\beta + \gamma QFS2]/D = (1-\alpha)(120-10) + [20 + \gamma 130]x0.9$
	$a7 = y4 + y7/D = (1-\alpha)(QSS1-KSS) + 0/D = (1-\alpha)(140-20) + 0/D$
	$a8 = y4 + y8/D = (1 - \alpha)(QSS1 - KSS) + [\beta + \gamma QSS2]/D = (1 - \alpha)(140 - 20) + [20 + \gamma 180]x0.9$
Son	
	b2 = Y2.1 + Y2.2/D = wL + wL/D = 60 + 0.9x60 = 114
	$b5 = Y3 + 0/D = \alpha(QFS1-KFS) + wL1'' + 0/D = \alpha(120-10) + 30 + 0/D$
	$b6 = Y3 + Y6/D = \alpha(QFS1-KFS) + wL1" + [(1-\gamma)QFS2 - \beta + wL2"]/D =$
	$\alpha(120-10) + 30 + [(1-\gamma)130 - 20 + 20]x0.9$
	$b7 = Y4 + Y7/D = \alpha(QSS1-KSS) + wL1" + [QSS2 + wL2"]/D = \alpha(140-20) + 30 + [180 + 20]x0.9$
	$b8 = Y4 + Y8/D = \alpha(QSS1-KSS) + wL1" + [(1-\gamma)QSS2 - \beta + wL2"]/D =$
	$\alpha(140-20) + 30 + [(1-\gamma)180 - 20 + 20]x0.9$

The values in Table 8.2 in the text are rounded to the nearest digit.

9. Survival Probabilities in Retirement

The purpose of these notes is to clarify some aspects of age transition probabilities and their application. The motivation is the analysis of Kotlikoff and Spivak (1981) and I shall attempt to suggest definitions which are consistent with the structure of the models they use.

Timing

Time in the analysis is discrete, the unit is one year. Year 1 is between time 0 and time 1; year two between t=1 and t=2 (see Figure 9.1). Two equations in Kotlikoff and Spivak (1981) set the timing of events

$$W_t = R(W_{t-1} - C_{t-1}) \qquad t = 1, 2, ..., T$$
 (1)

$$\sum_{t=0}^{T} P_t C_t R^{-t} = W_0 \qquad P_0 = 1$$
 (2)

Equation (1) is from Kotlikoff and Spivak (1981:A10) and it specifies consumption of year t-1 to occur at the beginning of year t; the remaining wealth is then compounded by R to time t. Equation (2) is from Kotlikoff and Spivak (1981:3) with $P_0=1$ added. The probability P_t is interpreted as the survival probability to time t. Accordingly, consumption C_t is occurring at time t; that is, at the beginning of the year t+1.



Figure 9.1. Time and survival probabilities

Probabilities

It is convenient to think of the transition process as starting at the retirement point and to define that point as time 0 (a person is "born" into retirement at "age" zero; again, his first year is between 0 and 1, his first "birthday" is at t=1). The transition probabilities are defined relatively to point 0.

The probability of death in the year t is g_t . Thus if 100 people are retiring at t = 0, 100 g_t of them can be expected to die in the year t. Note that in Figure 9.1, g_{t-1} is placed to the left of P_t .

If the event death (with the probability g_{t-1}) is realized in the year t, the person does not reach the time t. The probability of dying at any age up to t and including t is G_t

$$G_t = \sum_{j=0}^t g_j \tag{3}$$

If T, the number standing for the last age in the analysis, is large enough, it may be the last possible year of life; a person is sure to die by T+1. Then the probability of dying before reaching age T+1 is 1; $G_{T+1} = 1$.

The probability of survival is P_t . Again, $100P_t$ persons of a group of 100 retiring at t = 0 can be expected to survive to the point t.

$$P_{0} = 1$$

$$P_{1} = 1 - g_{0} = 1 - G_{1}$$

$$P_{t} = 1 - \sum_{j=0}^{t-1} g_{j} = 1 - G_{t}$$
(4)

If $G_{T+1} = 1$, $P_{T+1} = 0$; no one survives to age T+1. Given survival probabilities, probabilities of death can be calculated by first differences, $g_t = P_{t-1} - P_t$.

Life Expectancy

Life expectancy at "age" zero, e_0 , can be seen as the expected age of death or as the expected age of survival. The expected age of death is

$$e_0 = \sum_{t=0}^{T+1} g_t t$$
 (5)

The expected age of survival is

$$e_0 = \sum_{t=0}^T P_t \tag{6}$$

The equivalence of eqs. (5) and (6) is shown in (7)

$$\sum_{t=0}^{T} P_t = [1 - g_0] + [1 - (g_0 + g_1)] + \dots$$
$$= \sum_{t=1}^{T+1} g_t + \sum_{t=2}^{T+1} g_t + \sum_{t=3}^{T+1} g_t + \dots$$
$$= \sum_{t=0}^{T+1} g_t t$$
(7)

Life expectancy at age t is e_t . To compute it, define $g_t{}^j$ as the probability to die at j relative to time t (j > t)

$$g_t^j = g_j / P_t \tag{8}$$

and

$$e_t = \sum_{j=t+1}^{T+1} g_t^j j$$
 (9)

As $P_t < 1$ for t > 0,

$$e_t \geq \sum_{j=t+1}^{T+1} g_j j$$
 (10)

that is, life expectancy progresses with the age of the surviving person. (In Israel, life expectancy at birth is 74.4 and at age 60 it is 19--survival is then expected to age 79.)

Expected Utility

The expected utility is defined in Kotlikoff and Spivak (1981:1) as

$$EU = \sum_{t=0}^{T} P_t U(C_t) \tag{11}$$

In (11) U(C_t) is the utility of the individual at age t, if he or she survived to this age. In eq. (12), EU is expressed as a function of the g_t values.

$$EU = \sum_{t=1}^{T+1} g_t \sum_{j=0}^{t-1} U(C_j)$$
(12)

The second sum in (12) is the utility that an individual (dying at age t) accumulated up to the age t-1. The advantage of the formulation in (12) in defining the **expected** utility is that, unlike the P_t values in (11), the g_t probabilities sum to 1.

To prove the equivalence of (11) and (12) proceed as in (7):

$$\sum_{t=0}^{T} P_t U(C_t) = U_0 [1 - g_0] + U_1 [1 - (g_0 + g_1)] + \dots$$

$$= U_0 \sum_{t=1}^{T+1} g_t + U_1 \sum_{t=2}^{T+1} g_t + \dots$$

$$= g_1 U_0 + g_2 (U_0 + U_1) + \dots$$

$$= \sum_{t=1}^{T+1} g_t \sum_{i=0}^{t-1} U(C_t)$$
(13)

Inheritance

Dying at t, a person leaves as inheritance the part of the wealth that was not consumed up to t; that is, W_t as defined in (1). The expected value (not discounted) of the inheritance is

$$EI = \sum_{t=0}^{T+1} g_t W_t$$
 (14)

Once again, if T is the last possible surviving age, $W_{T+1}=0$.

A single person with a non-linear utility function and no annuity arrangements will almost always leave a positive inheritance. With annuities, the inheritance is the residual value of the wealth the insured left at death. Fair annuity maintains eq. (2). It will now be shown that the expected value of the inheritance in the case of fair annuity and for R = 1 is zero. The case of R > 1 is taken below.

For R = 1, eq. (1) is

$$W_t = W_0 - \sum_{j=0}^{t-1} c_j \qquad t = 1, 2, ..., T$$
 (15)

Eq. (14) can now be written as

$$EI = g_0 W_0 + \sum_{t=1}^{T+1} g_t [W_0 - \sum_{j=0}^{t-1} C_j]$$

= $W_0 - \sum_{t=1}^{T+1} g_t \sum_{j=0}^{t-1} C_j$ (16)

To prove that EI = 0 (remember, fair annuity) we have to show that the expression with the double summation in the right hand side of the second line of (16) is equal to W_0 ,

$$\sum_{t=1}^{T+1} g_t \sum_{j=0}^{t-1} C_j$$

$$= g_1 C_0 + g_2 (C_0 + C_1) + \dots + G_{T+1} (C_0 + C_1 + \dots + C_{T-1})$$

$$= C_0 \sum_{j=1}^{T+1} g_j + C_1 \sum_{j=2}^{T+1} g_{tj} + \dots$$

$$= \sum_{t=0}^{T} C_t [\sum_{j=t+1}^{T+1} g_j]$$

$$= \sum_{t=0}^{T} C_t [\sum_{j=0}^{T+1} g_j - \sum_{j=0}^{t} g_j]$$

$$= \sum_{t=0}^{T} C_t [1 - G_t]$$

$$= \sum_{t=0}^{T} C_t P_t = W_0 \qquad QED$$

For R > 1, the discounted value of the expected inheritance, with fair annuity, is zero. To show it, write the expected discounted inheritance as

$$EI = \sum_{t=0}^{T+1} g_t R^{-t} W_t$$
 (18)

Expanding, eq. (1) can be written as

$$W_t = R^t W_0 - \sum_{j=0}^{t-1} R^{t-j} C_j \qquad (t = 1, 2, ..., T)$$
(19)

Now eq. (18) is

$$EI = g_0 W_0 + \sum_{t=1}^{T+1} g_t R^{-t} [R^t W_0 - \sum_{j=0}^{t-1} R^{t-j} C_j]$$

= $\sum_{t=0}^{T+1} g_t W_0 - \sum_{t=1}^{T+1} g_t \sum_{j=0}^{t-1} R^{-j} C_j$ (20)

And again, as in eq. (17), the two terms in the second line of (20) are W_0 and the expected discounted inheritance is zero.

References

Becker, Gary S. (1991) A Treatise on the Family. Cambridge, MA: Harvard University Press.

- Kimhi, Ayal (1995a) "Specific human capital and the choice of successor in family farms." Chapter 10 in Intergenerational Transfers of Farmers in Different Institutional Environments, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal Kimhi, and Ramon Lopez.
- Kimhi, Ayal (1995b) "Optimal timing of farm transferal from parent to child." Chapter 18 in *Intergenerational Transfers* of Farmers in Different Institutional Environments, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal Kimhi, and Ramon Lopez.
- Kimhi, Ayal (1995c) "Optimal timing of transferring the family business to a heir." Chapter 11 in Intergenerational Transfers of Farmers in Different Institutional Environments, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal Kimhi, and Ramon Lopez.
- Kislev, Yoav, Zvi Lerman, and Pinhas Zusman (1991) "Recent Experience with Cooperative Farm Credit in Israel," *Economic Development and Cultural Change* 39: 773-89.
- Kotlikoff, Lawrence J., and Aviya Spivak (1981) "The Family as an Incomplete Annuities Market." *Journal of Political Economy* 89: 372-91.
- Nevo, Naomi (1995) "Inter-generational transfer of farms in Moshvei-Ovdim: an ethnographical enquiry." Chapter 3 in Intergenerational Transfers of Farmers in Different Institutional Environments, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal Kimhi, and Ramon Lopez.
- Pesquin, Claudio (1995) "A value assessment of intergenerational transfer of farms." Chapter 17 in Intergenerational Transfers of Farmers in Different Institutional Environments, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal Kimhi, and Ramon Lopez.
- Regev, Shuki (1995) "Farm succession in the moshav a legal examination." Chapter 2 in *Intergenerational Transfers of* Farmers in Different Institutional Environments, BARD Project IS-1845-90 Final Report by Yoav Kislev, Ayal

Kimhi, and Ramon Lopez. See also **Regev, Shuki** (1994) *Farm Succession - the Legal Aspect*. Working Paper No. 9406, The Center for Agricultural Economic Research, Rehovot, Israel (Hebrew).

- Shubik, Martin (1981) "Society, land, love or money: A strategic model of how to glue the generations together." *Journal of Economic Behavior and Organization* 2(4): 359-385.
- Zusman, Pinhas (1988) Individual Behavior and Social Choice in a Cooperative Settlement, The Theory and Practice of the Israeli Moshav. Jerusalem, Israel: Magnes Press.